FORM TP 02134010/SPEC TEST CODE 02134010/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

SPECIMEN PAPER

UNIT 1 – PAPER 1

2 hours

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3

Each section consists of 5 questions.
The maximum mark for each Module is 40.
The maximum mark for this examination is 120.
This examination consists of 7 printed pages.

INSTRUCTIONS TO CANDIDATES

1. DO NOT open this examination paper until instructed to do so.
2. Answer ALL questions from the THREE sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct three significant figures.

Examination Materials:

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer ALL questions.

1. (a) Express $\frac{2x}{x-2}$ in the form $p + \frac{q}{x-2}$

stating clearly the values of the constants p and q. [3 marks]

(b) Given that $hx^3 - 12x^2 - x + 3 \equiv (2x - 1)(2x + 1)(x - k)$, find the values of the constants h and k. [3 marks]

(c) Using the fact that $\sum_{r=1}^{n} kr = k \sum_{r=1}^{n} r = k\left[\frac{n(n+1)}{2}\right]$, where k is a constant, and that $\sum_{r=1}^{n} k = k \sum_{r=1}^{n} 1 = kn$, find, in simplest form, the value of $\sum_{r=1}^{n}(6r + 5)$. [3 marks]

Total 9 marks

2. Find the real values of x which satisfy the equations

(a) $|2x - 3|^2 - 6|2x - 3| + 5 = 0$ [5 marks]

(b) $2^{3-5x} = \left(\frac{1}{64}\right)^{x-1}$. [4 marks]

Total 9 marks

3. Find the range of values of x for which

(a) $\frac{x}{x+1} > 0$ [4 marks]

(b) $(2x + 1)^2 \leq 9$. [4 marks]

Total 8 marks

4. A pair of simultaneous equations is given by

$$p^2x - 4y = 8$$

$$8x - 2y = p$$

where $p \in \mathbb{R}$.

(a) Find the value of p for which the equations have an infinite number of solutions. [4 marks]

(b) Find the solutions for this value of p. [2 marks]

Total 6 marks

5. The functions, f and g, are defined by

$$f: x \to \frac{x^2}{1-x^2}, x^2 \neq 1;$$

$$g: x \to \frac{1}{2}x - 3$$

(a) Explain clearly why f is NOT one-to-one [3 marks]

(b) Find, and express in simplest form, $gf(x)$. [5 marks]

Total 8 marks

SECTION B (MODULE 2)

Answer ALL questions.

6. (a) (i) Find the coordinates of the point D on the line segment joining A $(1, -4)$ and B $(-3, 12)$ such that the ratio $AD : DB$ is $3 : 1$. [3 marks]

(ii) Find the equation of the line through D parallel to the line $y - 2x = 3$. [3 marks]

(b) Find the equation of the circle with the centre $(-1, 1)$ and radius $\sqrt{2}$ units, in the form

$$x^2 + y^2 + ax + by + c = 0$$

where $a, b, c \in R$. [2 marks]

Total 8 marks

7. (a) State, in surd form, the values of $\tan 30°$ and $\tan 60°$. [1 mark]

(b) Express $\tan (A - B)$ in terms of $\tan A$ and $\tan B$. [1 mark]

(c) Hence, or otherwise, without the use of calculators, show that

$$\tan 15° = 2 - \sqrt{3}.$$ [3 marks]

(d) Given that $\tan (\theta + 45) = x$, express $\tan \theta$ in terms of x. [3 marks]

Total 8 marks

8. (a) Solve $6 \sin^2 x - \cos x - 4 = 0$ for $0 \leq x \leq \pi/2$. [6 marks]

(b) Find the range of values of $k \in R$ for which the equation $x^2 + kx + 1 = 0$ has complex roots. [2 marks]

Total 8 marks

9. Find ALL the complex numbers $x + yi$ such that

$$(x + yi)^2 = -3 + 4i.$$

Total 8 marks

10. (a) Find the values of $t \in \mathrm{R}$ such that the vectors

$$u = 3i + 2tj$$
$$v = -2i + 3tj$$

are perpendicular.

[4 marks]

(b) The position vectors of points A and C relative to the origin, O, are $4i + j$ and $i + 7j$ respectively. Find $\cos A\hat{O}C$.

[4 marks]

Total 8 marks

SECTION C (MODULE 3)

Answer ALL questions.

11. (a) Find $\lim_{x \to 0} \frac{(x+1)^2 - (x-1)^2}{2x}$ [4 marks]

(b) Given that $\lim_{x \to 0} \frac{\sin x}{x} = 1$,

evaluate

$$\lim_{x \to 0} \frac{\sin 4x}{x}.$$

[3 marks]

Total 7 marks

12. The function f is defined by

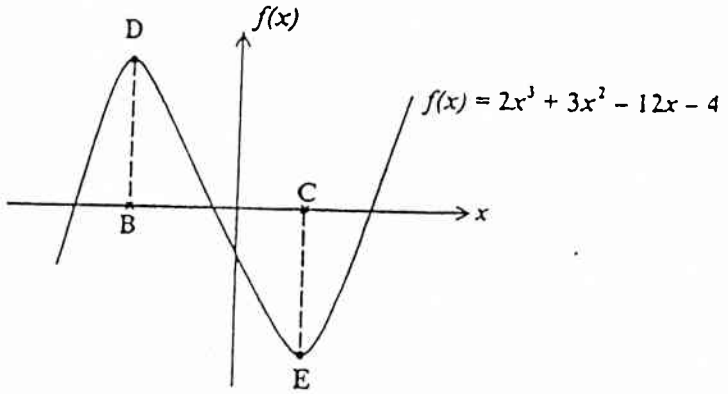
$$f(x) = (x^2 - 4)^3.$$

(a) Find $f'(x)$, simplifying your result as far as possible. [3 marks]

(b) Determine the values of x for which $f(x)$ is stationary. [4 marks]

Total 7 marks

13. (a) The diagram below is a sketch of $f(x) = 2x^3 + 3x^2 - 12x - 4$ showing its stationary points, D and E.



Find the coordinates of the points labelled

(i) B
(ii) C.

[5 marks]

(b) Determine the range of values of x for which the function

$$f(x) = 2x^3 + 3x^2 - 12x - 4$$

is increasing.

[3 marks]

Total 8 marks

14. The function $f(x)$ is such that

$$f'(x) = 9x^2 + 4x + c,$$

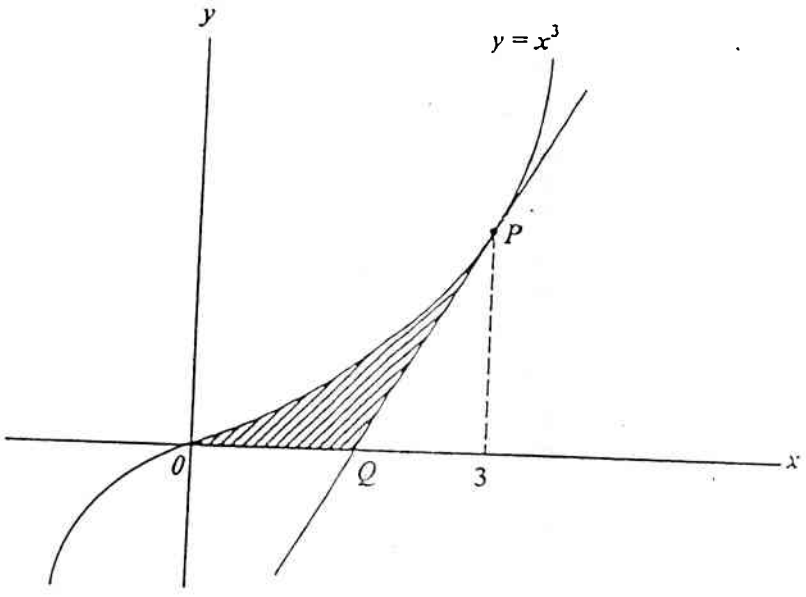
where c is a constant.

Given that $f(2) = 14$ and $f(3) = 74$,

find the value of $f(4)$

Total 8 marks

15. The diagram below, **not drawn** to scale is a sketch of the curve $y = x^3$ and the tangent PQ to the curve at $P(3, 27)$.



(a) Find

(i) the equation of the tangent, PQ [4 marks]
(ii) the coordinates of Q. [1 mark]

(b) Calculate the area of the shaded region in the diagram. [5 marks]

Total 10 marks

END OF TEST