FORM TP 02134020/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

SPECIMEN PAPER

UNIT 1 – PAPER 02

2 hours

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3

Each section consists of 2 questions.
The maximum mark for each Module is 40.
The maximum mark for this examination is 120.
This examination consists of 6 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct three significant figures.

Examination Materials:

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer ALL questions.

1. (a) (i) Construct a table for the function, $f(x) = x^3 - 3x + 2$ for $x = 0, 0.5, 1.0, 1.5, 2.0$. **[2 marks]**

(ii) Using a scale of 5 cm to represent 1 unit on the domain and 2 cm to represent 1 unit on the codomain, draw the graph of $f(x)$, $0 \leq x \leq 2$. **[3 marks]**

(iii) On the same graph, draw $g(x) = x - 1$ for $0 \leq x \leq 2$. **[1 mark]**

(iv) Estimate to 1 decimal place,

a) the value(s) of x for which $f(x) = g(x)$ **[1 mark]**

b) the range of values of x for which $f(x) < g(x)$. **[1 mark]**

(v) Use the information from your graph in (ii) above to obtain a linear factor of $f(x)$. **[2 marks]**

(b) Factorise completely $x^3 - 3x + 2$. **[5 marks]**

(c) The roots of the quadratic equation $x^2 - 3x - 1 = 0$ are α and β.

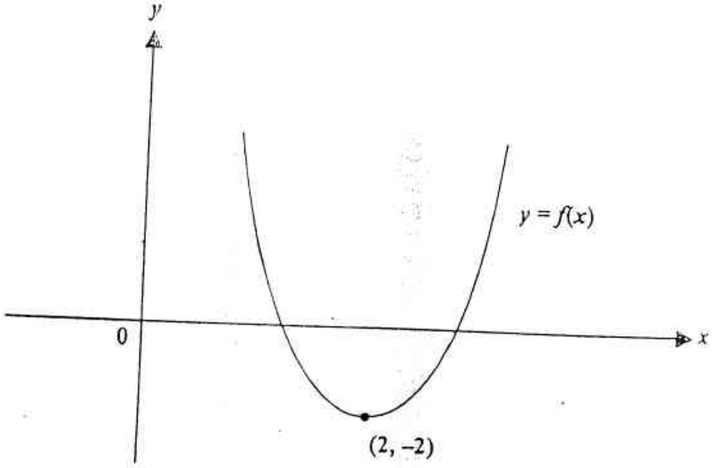
Without solving the equation, obtain the equation whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$. **[5 marks]**

Total 20 marks

2. (a) Given that the sum of the first n terms of the series $\sum_{r=1}^{n}(6r+5)$ is $3n^2 + 8n$, calculate the first five partial sums of the given series. **[2 marks]**

(b) Prove by Mathematical Induction that $\sum_{r=1}^{n}(6r+5) = n(3n+8)$. **[9 marks]**

(c) The diagram below, not drawn to scale, shows the graph of $y = f(x)$ which has a minimum at $(2, -2)$.



Copy this diagram and on the same axes sketch the graphs of:

(i) $y = f(x - 1)$ [3 marks]

(ii) $y = f(x) + 3$ [3 marks]

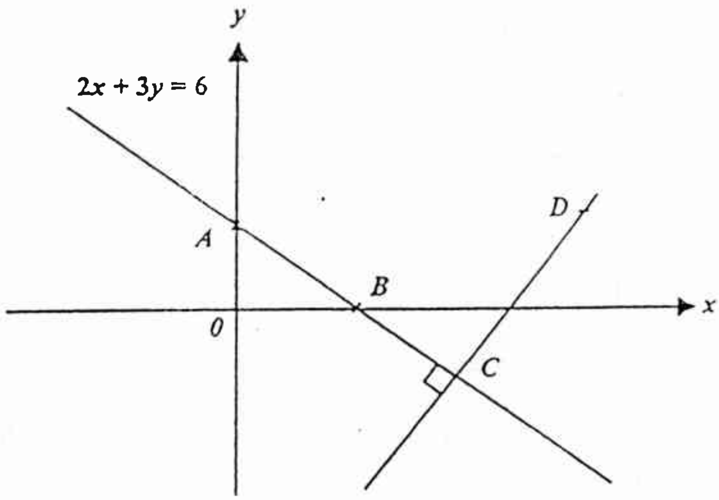
(iii) $y = |f(x)|$ [3 marks]

Total 20 marks

SECTION B (MODULE 2)

Answer ALL questions.

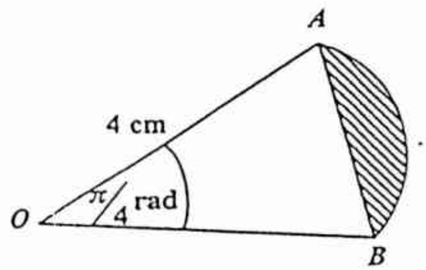
3. (a) In the diagram shown below, not drawn to scale, the line $2x + 3y = 6$ meets the y-axis at A and the x-axis at B. C is the point on AB produced such that B is the mid-point of AC.



(i) Find the coordinates of A, B and C. [6 marks]

(ii) Find the equation of the line CD through C perpendicular to AB. [3 marks]

(b) The diagram shown below, not drawn to scale, is a sketch of a sector of a circle, centre O and radius 4 cm. Angle AOB measures $\frac{\pi}{4}$ radians.



(i) Show that the area of the shaded region is $2\left(\pi - 2\sqrt{2}\right)$ cm^2. [7 marks]

(ii) Using the cosine rule, show the length of the chord AB is $4\sqrt{\left(2-\sqrt{2}\right)}$ cm. [4 marks]

Total 20 marks

4. (a) Solve $\sin\theta + \sin 2\theta + \sin 3\theta = 0$ for $0 \le \theta \le \pi$. [7 marks]

(b) (i) Express the complex number $\dfrac{4-2i}{1-3i}$ in the form of $a + bi$ where a and b are real numbers. [4 marks]

(ii) Show that the argument of the complex number in (b)(i) above is $\dfrac{\pi}{4}$. [1 mark]

(c) The position vectors of two points A and B are

$-2\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{j}$ respectively.

Find

(i) the unit vector in the direction of $\overrightarrow{OB}$ [1 mark]

(ii) the position vector of the point C on $\overrightarrow{OB}$ produced such that

$$|\overrightarrow{OC}| = |\overrightarrow{OA}|$$ [4 marks]

(iii) the position vector of the point D on AB produced such that $\overrightarrow{AD} = 2\,\overrightarrow{AB}$. [3 marks]

Total 20 marks

SECTION C (MODULE 3)

Answer ALL questions.

5. (a) Show that $\lim_{h \to 0} \dfrac{h}{\sqrt{x+h} - \sqrt{x}} = 2\sqrt{x}$. [5 marks]

(b) (i) Given that $f(x) = x^3 - 5x^2 + 3x$, show that $f(x) = 0$ possesses a root in the interval $\left[\frac{1}{2}, 1\right]$. [3 marks]

(ii) By considering suitable values of x greater than 1, show that there is another root of $f(x) = 0$ greater than 1. [4 marks]

(c) If $y = \dfrac{x}{1 + x^2}$, show that

$$\frac{d^2y}{dx^2} = \frac{2y\left(x^2 - 3\right)}{\left(1 + x^2\right)^2}$$

[8 marks]

Total 20 marks.

6. (a) Using the trapezium rule with 5 ordinates, evaluate

$$\int_0^1 \frac{1}{\left(x^2 + 1\right)^{3/2}}\, dx,$$

giving your answer correct to 3 significant figures. [5 marks]

(b) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 8x + 5.$$

The curve passes through the point (0, 3).

(i) Find the equation of the curve. [3 marks]

(ii) Find the coordinates of the two stationary points and identify the nature of each. [7 marks]

(c) (i) Sketch the curve $y = 9 - x^2$, stating the coordinates of the intersections with the axes. [2 marks]

(ii) The finite region bounded by the curve, the y-axis, and the x-axis is denoted by R.

Find the volume of the solid generated when R is rotated completely about the y-axis, giving your answer in terms of π. [3 marks]

Total 20 marks

END OF TEST