FORM TP 2005254

TEST CODE **02134032**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 03/B

$1\frac{1}{2}$ hours

20 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 1 question.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer this question.

1. (a) Given that $2x^2 + 8x + 11 = 2(x + h)^2 + k$ for all values of x, find the value of EACH of the constants h and k. **[5 marks]**

(b) (i) If $p, q, r, s \in \mathbf{R}$, use the fact that $(p - q)^2 \geq 0$ to show that $p^2 + q^2 \geq 2\,pq$. **[2 marks]**

(ii) Deduce that if $p^2 + q^2 = 1$, then $pq \leq \frac{1}{2}$. **[1 mark]**

(c) A club bakes and sells x cakes, making a profit, in dollars, that is modelled by the function $f(x) = x^2 - 10x$.

(i) Sketch the graph of the function $f(x) = x^2 - 10x$. **[8 marks]**

(ii) From your graph, determine

a) the LEAST number of cakes sold for which a profit is realised **[2 marks]**

b) the GREATEST possible loss in dollars **[1 mark]**

c) the number of cakes for which the GREATEST possible loss occurs. **[1 mark]**

Total 20 marks

Section B (Module 2)

Answer this question.

2. (a) The straight line through the point P (4, 3) is perpendicular to $3x + 2y = 5$ and meets the given line at N.

Find

(i) the coordinates of N **[6 marks]**

(ii) the length of the line-segment PN. **[2 marks]**

(b) The table below presents data collected on the movement of the tide at various times after midnight on a particular day.

Tide Movement	**Time After Midnight (t hours)**	**Height (h metres)**
High	0	12
Low	6	2
High	12	12
Low	18	2

The height, h metres, can be modelled by a function of the form $h = p\cos(qt)^\circ + 7$ where t is the time in hours after midnight. Use the data from the table to find the values of p and q. **[12 marks]**

Total 20 marks

Section C (Module 3)

Answer this question.

3. (a) (i) Find $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$. **[3 marks]**

(ii) Determine the real values of x for which the function

$$f(x) = \frac{3x - 1}{x^2 - x - 2}$$

is continuous. **[3 marks]**

(b) Differentiate with respect to x, **from first principles**, the function $x^2 + 2x$. **[5 marks]**

(c) Initially, the depth of water in a tank is 32 m. Water drains from the tank through a hole cut in the bottom. At t minutes after the water begins draining, the depth of water in the tank is x metres. The depth of the water changes, with respect to time t, at the rate equal to $(-2t - 4)$.

(i) Find an expression for x in terms of t. **[5 marks]**

(ii) Hence, determine how long it takes for the water to drain completely from the tank. **[4 marks]**

Total 20 marks

END OF TEST