FORM TP 2004247

TEST CODE 02134032

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 03/2

1 $\frac{1}{2}$ hours

21 MAY 2004 (p.m.)

4247

This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 1 question.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

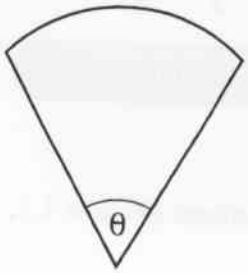
Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 1.1)

Answer this question.

1. (a) The expressions $x^3 - 7x + 6$ and $x^3 - x^2 - 4x + 24$ have the same remainder when divided by $x - p$. Find the possible value(s) of the constant p. **[7 marks]**

(b) The figure below **(not drawn to scale)** shows a piece of wire 40 cm long formed into the shape of a sector of a circle of radius r cm, and angle θ radians.



(i) Write down an expression for the perimeter of the sector in terms of r and θ, and hence, show that $\theta = \dfrac{40 - 2r}{r}$. **[3 marks]**

(ii) Show that the area, A cm², of the sector is given by $A = 20r - r^2$. **[2 marks]**

(c) Solve the pair of simultaneous equations

$$x^2 + xy = 3$$

$$y - 3x = -1 .$$

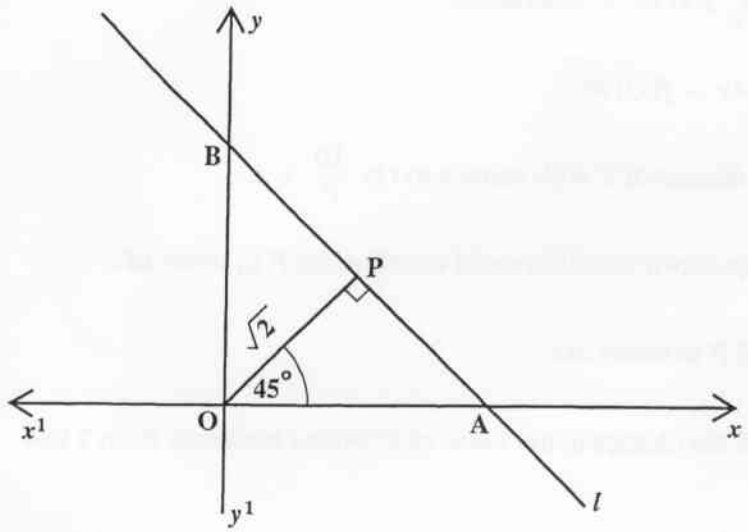
[8 marks]

Total 20 marks

SECTION B (MODULE 1.2)

Answer this question.

2. (a) In the diagram below **(not drawn to scale)**, the line l cuts the positive x and y axes at the points A and B respectively. The perpendicular from the origin O meets l at P, $OP = \sqrt{2}$ units and $P\hat{O}A = 45°$.



(i) Find the coordinates of P. **[3 marks]**

(ii) Show that the equation of l is $x + y = 2$. **[3 marks]**

(b) (i) Express $\sin 4\theta$ in terms of $\sin 2\theta$ and $\cos 2\theta$. **[1 mark]**

(ii) Hence, solve the equation $\sin 4\theta = \cos 2\theta$ for $0 \leq \theta \leq \pi$. **[5 marks]**

(c) The roots of the quadratic equation $x^2 - 4x + c = 0$ are the complex numbers $2 + i$ and $2 - i$. Find the value of the constant c. **[2 marks]**

(d) The position vectors of two points A and B are $2\mathbf{i} + 3\mathbf{j}$ and $3\mathbf{i} - 8\mathbf{j}$ respectively. D is the midpoint of AB and the point E divides OD in the ratio 2:3. Find the position vector of E. **[6 marks]**

Total 20 marks

SECTION C (MODULE 1.3)

Answer this question.

3. (a) Evaluate $\lim_{x \to -2} \frac{x^3 - 4x}{x^2 + 5x + 6}$. **[6 marks]**

(b) Differentiate, with respect to t, the function $\frac{t^4 - 2t^2 + 1}{2t^2}$. **[4 marks]**

(c) Given that $\int_0^4 f(x)\,dx = 6$, evaluate

$$\int_0^4 (4x - f(x))\,dx.$$ **[3 marks]**

(d) The rate of change of P with respect to t is $\frac{10}{t^2} + t$.

(i) Write down the differential equation for P in terms of t. **[1 mark]**

(ii) Find P in terms of t. **[3 marks]**

(iii) Find the change in the value of P when t increases from 2 to 4. **[3 marks]**

Total 20 marks

END OF TEST