Circle Geometry

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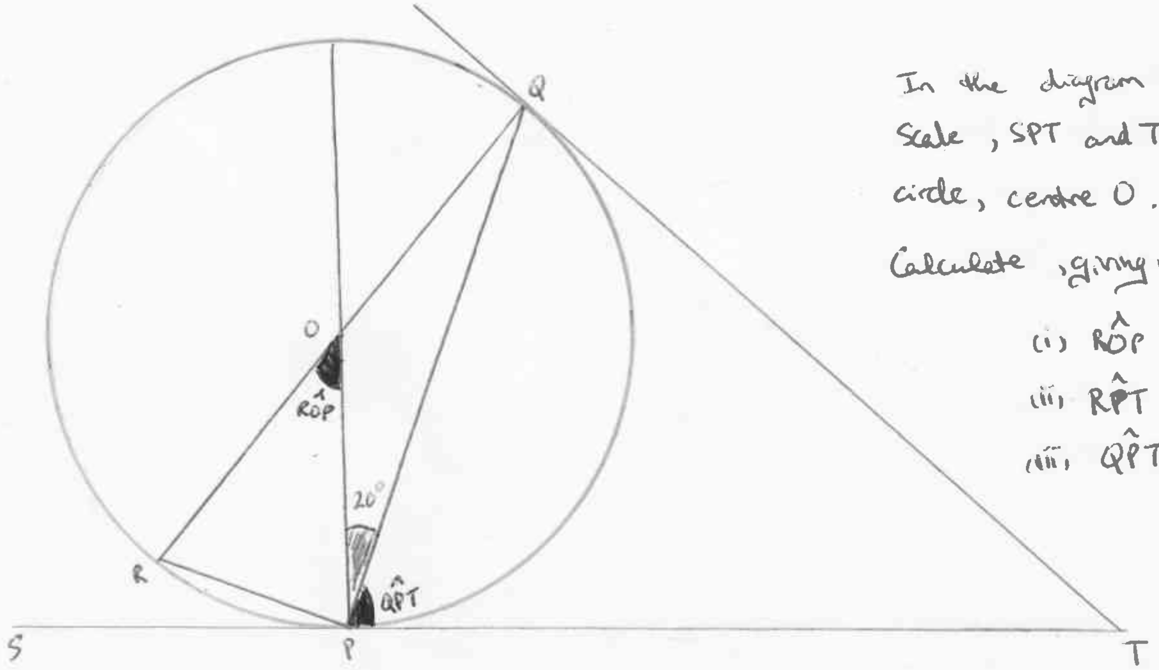
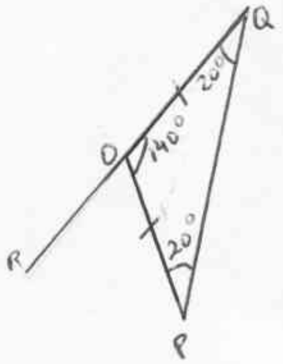


In the diagram above, not drawn to scale, SPT and TQ are tangents to the circle, centre O. Angle $OPQ = 20^\circ$.
Calculate, giving reasons:

(i) $R\hat{O}P$
(ii) $R\hat{P}T$
(iii) $Q\hat{P}T$

Consider ΔPOQ. $|OP| = |OQ|$ since OP and OQ are radii of the circle.
So ΔPOQ is an isosceles triangle.

Thus, $O\hat{P}Q = O\hat{Q}P = 20^\circ$ [Base angles of isosceles are equal].



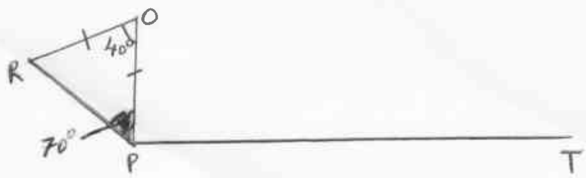
$P\hat{O}Q = 180^\circ - [20^\circ + 20^\circ] = 140^\circ$ [Sum of angles in triangle add to 180°].

$R\hat{O}P = 180^\circ - P\hat{O}Q$ [Angles on straight line add up to 180°].

$\Rightarrow R\hat{O}P = 180^\circ - 140^\circ = 40^\circ$.

Alternatively, $R\hat{O}P = 2\,R\hat{Q}P$ [Angle subtended at centre is twice the angle subtended at chord, standing on arc RP]

So $R\hat{O}P = 2(20^\circ) = 40^\circ$.



$R\hat{P}T = R\hat{P}O + O\hat{P}T$

Now, $\Delta R\hat{O}P$ is isosceles triangle, as lengths $|OR| = |OP|$.

So $R\hat{P}O = \dfrac{180^\circ - 40^\circ}{2} = 70^\circ$

$O\hat{P}T = 90^\circ$ [Angle subtended between radius OP and tangent SPT is 90°]

Thus, $R\hat{P}T = 70^\circ + 90^\circ = 160^\circ$.

$Q\hat{P}T = 90^\circ - 20^\circ = 70^\circ$, since $O\hat{P}T = 90^\circ$ and $O\hat{P}Q + Q\hat{P}T = O\hat{P}T$.