FORM TP 2007246

TEST CODE **02134010**

MAY/JUNE 2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 01

2 hours

18 MAY 2007 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. Given that $x - 1$ is a factor of the function $f(x) = x^3 + px^2 - x - 2,\ p \in \mathbf{R}$, find

   (a) the value of p **[2 marks]**

   (b) the remaining factors of $f(x)$. **[4 marks]**

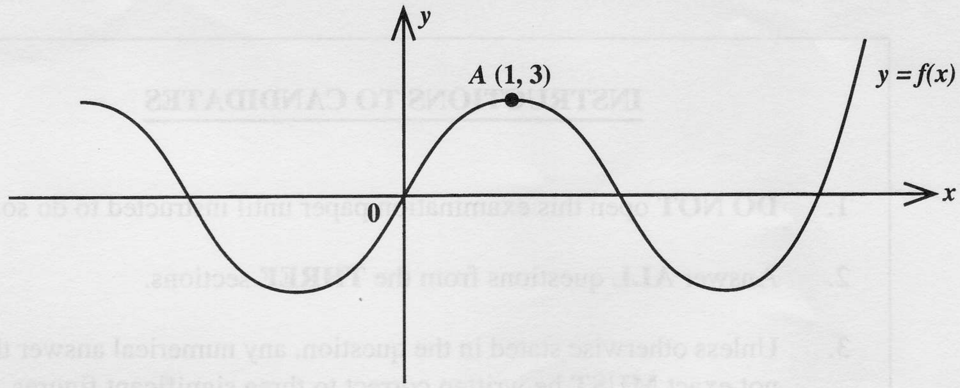
Total 6 marks

2. (a) Solve, for x, the equation $(3^x)^2 = 27^{(x-2)}$. **[4 marks]**

   (b) Express $\dfrac{5-\sqrt{3}}{2+\sqrt{3}}$ in the form $x + y\sqrt{3}$ where $x, y \in \mathbf{Z}$. **[5 marks]**

Total 9 marks

3. (a) The diagram below **(not drawn to scale)** shows the graph $y = f(x)$ which has a local maximum point at A (1, 3).



Determine the coordinates of the maximum point on each of the following graphs.

   (i) $y = f(x) - 2$ **[2 marks]**

   (ii) $y = f(x + 3)$. **[2 marks]**

   (b) The function, f, is defined on $\mathbf{R}$ by $f : x \rightarrow 3x - 2$.

   (i) Show that f is one-to-one. **[2 marks]**

   (ii) Hence, or otherwise, find the value of $x \in \mathbf{R}$ for which $f(f(x + 3)) = f(x - 3)$. **[4 marks]**

Total 10 marks

4. (a) Solve $|x - 4| - 6 > 0$ for **all** $x \in \mathbf{R}$. [5 marks]

(b) Find the real numbers u, v and w such that $-3x^2 - x + 2 \equiv u(x + v)^2 + w$. [3 marks]

Total 8 marks

5. Solve the following pair of simultaneous equations.

$$x^2 + xy = 2$$
$$y + 3x = 5$$

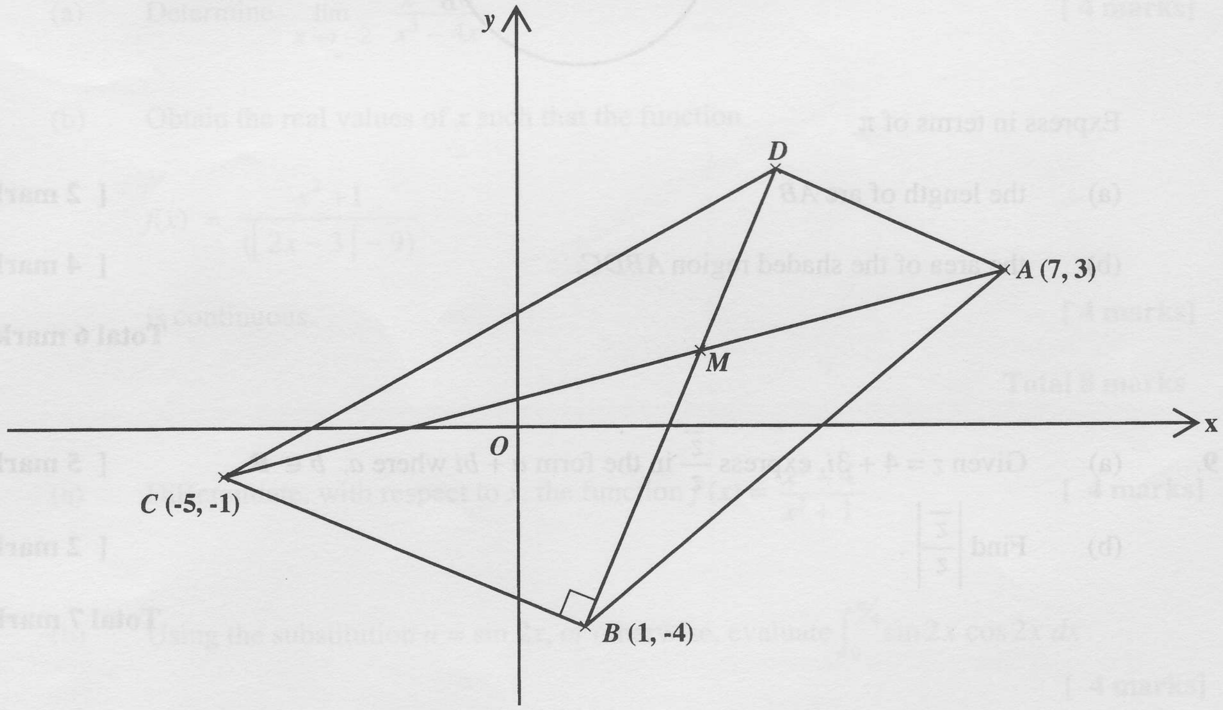
[7 marks]

Total 7 marks

Section B (Module 2)

Answer ALL questions.

6. In the diagram below (**not drawn to scale**), the points $A(7, 3)$, $B\ (1, -4)$, $C\ (-5, -1)$ are three vertices of a quadrilateral $ABCD$. The line BD is perpendicular to BC and M is the point of intersection of the lines AC and BD.



(a) Find the equation of

(i) the line AC [3 marks]

(ii) the line BD. [3 marks]

(b) Hence, find the coordinates of M. [3 marks]

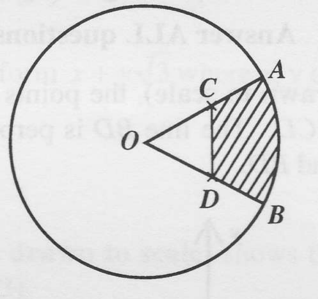
Total 9 marks

7. (a) Express $\cos\ \theta - \sin\ \theta$ in the form $R\cos(\theta + \alpha)$ where $R, \alpha \in \mathbf{R}$, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **[5 marks]**

(b) Hence, find the general solution of $\cos\theta - \sin\theta = 1$. **[3 marks]**

Total 8 marks

8. The circle in the diagram below, **not drawn to scale**, has centre O and the acute angle $AOB = \dfrac{\pi}{6}$ radians. $OA = 6$ cm and C and D are the midpoints of OA and OB respectively.



Express in terms of π

(a) the length of arc AB **[2 marks]**

(b) the area of the shaded region $ABDC$. **[4 marks]**

Total 6 marks

9. (a) Given $z = 4 + 3i$, express $\dfrac{\overline{z}}{z}$ in the form $a + bi$ where $a,\ b \in \mathbf{R}$. **[5 marks]**

(b) Find $\left|\dfrac{\overline{z}}{z}\right|$. **[2 marks]**

Total 7 marks

10. (a) The position vectors of points A and B with respect to an origin O are given by $\overrightarrow{OA} = 3\boldsymbol{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} - 4\mathbf{j}$. Find

(i) $\overrightarrow{AB}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$ **[2 marks]**

(ii) the magnitude of $\overrightarrow{AB}$ **[2 marks]**

(iii) the position vector of the point M that divides AB **internally** in the ratio 1:2. **[3 marks]**

(b) Determine whether $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. **[3 marks]**

Total 10 marks

Section C (Module 3)

Answer ALL questions.

11. (a) Determine $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x^3 - 4x}$. **[4 marks]**

(b) Obtain the real values of x such that the function

$$f(x) = \frac{x^2 + 1}{(|2x - 3| - 9)}$$

is continuous. **[4 marks]**

Total 8 marks

12. (a) Differentiate, with respect to x, the function $f(x) = \dfrac{x^2 - 4}{x^3 + 1}$. **[4 marks]**

(b) Using the substitution $u = \sin 2x$, or otherwise, evaluate $\int_0^{\pi/4} \sin 2x \cos 2x \, dx$. **[4 marks]**

Total 8 marks

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13. The curve $y = px^3 + qx + r$ passes through the origin O and the point $P(1, 2)$. The gradient of the curve at P is equal to 8.

(a) Calculate the values of the constants p, q and r. **[6 marks]**

(b) Obtain the equation of the normal to the curve at P. **[2 marks]**

Total 8 marks

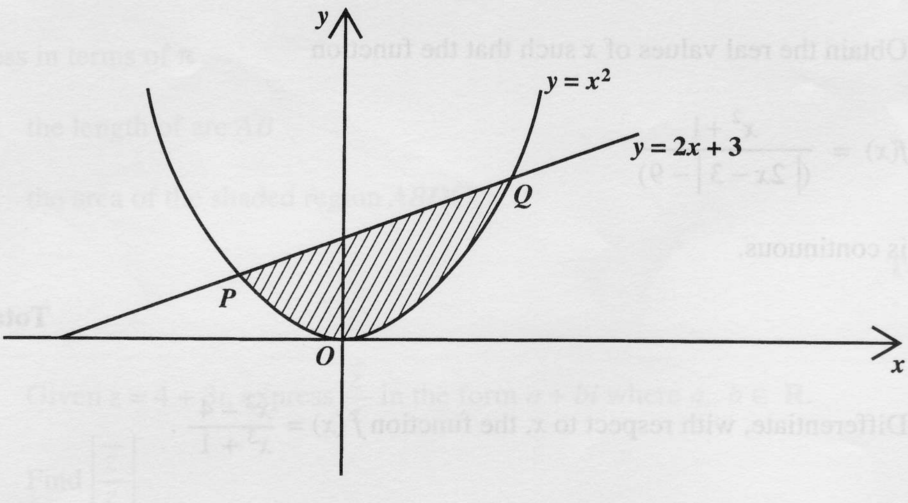
14. For the function $f : x \rightarrow 12x - x^3$, determine

(a) the stationary points **[4 marks]**

(b) the nature of EACH of the stationary points. **[3 marks]**

Total 7 marks

15. In the diagram below **(not drawn to scale)**, the line $y = 2x + 3$ cuts the curve $y = x^2$ at the points P and Q.



(a) Determine the coordinates of P and Q. **[4 marks]**

(b) Calculate the area of the shaded portion POQ shown in the diagram above. **[5 marks]**

Total 9 marks

END OF TEST