CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

ALGEBRA, GEOMETRY AND CALCULUS

2 ½ hours

21 MAY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

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SECTION A (Module 1)

Answer BOTH questions.

1. (a) The roots of the cubic equation $x^3 + 3px^2 + qx + r = 0$ are 1, –1 and 3. Find the values of the real constants p, q and r. **[7 marks]**

(b) Without using calculators or tables, show that

(i) $\dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = 2 + \sqrt{3}$ **[5 marks]**

(ii) $\dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} + \dfrac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = 4.$ **[5 marks]**

(c) (i) Show that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2), \quad n \in \hat{N}.$ **[5 marks]**

(ii) Hence, or otherwise, evaluate

$$\sum_{r=31}^{50} r(r+1).$$ **[3 marks]**

Total 25 marks

2. (a) The roots of the quadratic equation

$2x^2 + 4x + 5 = 0$ are α and β.

Without solving the equation

(i) write down the values of $\alpha + \beta$ and $\alpha\beta$ **[2 marks]**

(ii) calculate

a) $\alpha^2 + \beta^2$ **[2 marks]**

b) $\alpha^3 + \beta^3$ **[4 marks]**

(iii) find a quadratic equation whose roots are α^3 and β^3. **[4 marks]**

(b) (i) Solve for x the equation $x^{1/3} - 4x^{-1/3} = 3$. **[5 marks]**

(ii) Find x such that $\log_5 (x + 3) + \log_5 (x - 1) = 1$. **[5 marks]**

(iii) Without the use of calculators or tables, evaluate

$$\log_{10}\left(\frac{1}{2}\right) + \log_{10}\left(\frac{2}{3}\right) + \log_{10}\left(\frac{3}{4}\right) + \ldots + \log_{10}\left(\frac{8}{9}\right) + \log_{10}\left(\frac{9}{10}\right).$$

[3 marks]

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The lines $y = 3x + 4$ and $4y = 3x + 5$ are inclined at angles α and β respectively to the x-axis.

(i) State the values of $\tan \alpha$ and $\tan \beta$. **[2 marks]**

(ii) Without using tables or calculators, find the tangent of the angle between the two lines. **[4 marks]**

(b) (i) Prove that $\sin 2\theta - \tan \theta \cos 2\theta = \tan \theta$. **[3 marks]**

(ii) Express $\tan \theta$ in terms of $\sin 2\theta$ and $\cos 2\theta$. **[2 marks]**

(iii) Hence show, without using tables or calculators, that $\tan 22.5° = \sqrt{2} - 1$. **[4 marks]**

(c) (i) Given that A, B and C are the angles of a triangle, prove that

a) $\sin \dfrac{A+B}{2} = \cos \dfrac{C}{2}$ **[3 marks]**

b) $\sin B + \sin C = 2 \cos \dfrac{A}{2} \cos \dfrac{B-C}{2}$. **[2 marks]**

(ii) Hence, show that

$\sin A + \sin B + \sin C = 4 \cos \dfrac{A}{2} \cos \dfrac{B}{2} \cos \dfrac{C}{2}$. **[5 marks]**

Total 25 marks

4. (a) In the Cartesian plane with origin O, the coordinates of points P and Q are $(-2, 0)$ and $(8, 8)$ respectively. The midpoint of PQ is M.

(i) Find the equation of the line which passes through M and is perpendicular to PQ. **[8 marks]**

(ii) Hence, or otherwise, find the coordinates of the centre of the circle through P, O and Q. **[9 marks]**

(b) (i) Prove that the line $y = x + 1$ is a tangent to the circle $x^2 + y^2 + 10x - 12y + 11 = 0$. **[6 marks]**

(ii) Find the coordinates of the point of contact of this tangent to the circle. **[2 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Find $\lim_{x \to 3} \dfrac{x^3 - 27}{x^2 + x - 12}$. **[4 marks]**

(b) A chemical process is controlled by the function

$$P = \frac{u}{t} + vt^2, \text{ where } u \text{ and } v \text{ are constants.}$$

Given that $P = -1$ when $t = 1$ and the rate of change of P with respect to t is -5 when $t = \frac{1}{2}$, find the values of u and v. **[6 marks]**

(c) The curve C passes through the point $(-1, 0)$ and its gradient at any point (x, y) is given by

$$\frac{dy}{dx} = 3x^2 - 6x.$$

(i) Find the equation of C. **[3 marks]**

(ii) Find the coordinates of the stationary points of C and determine the nature of EACH point. **[7 marks]**

(iii) Sketch the graph of C and label the x-intercepts. **[5 marks]**

Total 25 marks

6. (a) Differentiate with respect to x

(i) $x\sqrt{2x-1}$ **[3 marks]**

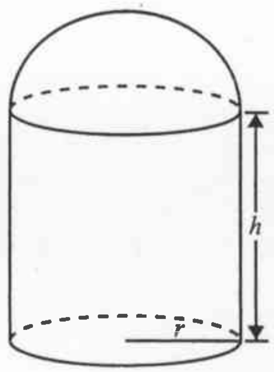
(ii) $\sin^2(x^3+4)$. **[4 marks]**

(b) (i) Given that $\int_1^6 f(x)\,dx = 7$, evaluate $\int_1^6 [2 - f(x)]\,dx$. **[3 marks]**

(ii) The area under the curve $y = x^2 + kx - 5$, above the x-axis and bounded by the lines $x = 1$ and $x = 3$, is $14\frac{2}{3}$ units².

Find the value of the constant k. **[4 marks]**

(c) The diagram below **(not drawn to scale)** represents a can in the shape of a closed cylinder with a hemisphere at one end. The can has a volume of 45π units³.



(i) Taking r units as the radius of the cylinder and h units as its height, show that,

a) $h = \frac{45}{r^2} - \frac{2r}{3}$ **[3 marks]**

b) $A = \frac{5\pi r^2}{3} + \frac{90\pi}{r}$, where A units is the external surface area of the can. **[3 marks]**

(ii) Hence, find the value of r for which A is a minimum and the corresponding minimum value of A. **[5 marks]**

[Volume of a sphere = $\frac{4}{3}\pi r^3$, surface area of a sphere = $4\pi r^2$.]

[Volume of a cylinder = $\pi r^2 h$, curved surface area of a cylinder = $2\pi r h$.]

Total 25 marks

END OF TEST