TEST CODE **02134020**

FORM TP 2009234

MAY/JUNE 2009

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

ALGEBRA, GEOMETRY AND CALCULUS

2 ½ hours

20 MAY 2009 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) **Without the use of tables or a calculator**, simplify $\sqrt{28} + \sqrt{343}$ in the form $k\sqrt{7}$, where k is an integer. **[5 marks]**

(b) Let x and y be positive real numbers such that $x \neq y$.

(i) Simplify $\dfrac{x^4 - y^4}{x - y}$. **[6 marks]**

(ii) Hence, or otherwise, show that

$$(y + 1)^4 - y^4 = (y + 1)^3 + (y + 1)^2\, y + (y + 1)\, y^2 + y^3.$$ **[4 marks]**

(iii) Deduce that $(y + 1)^4 - y^4 < 4(y + 1)^3$. **[2 marks]**

(c) Solve the equation $\log_4 x = 1 + \log_2 2x,\ x > 0$. **[8 marks]**

Total 25 marks

2. (a) The roots of the quadratic equation

$$2x^2 + 4x + 5 = 0 \text{ are } \alpha \text{ and } \beta.$$

Without solving the equation, find a quadratic equation with roots $\dfrac{2}{\alpha}$ and $\dfrac{2}{\beta}$.

[6 marks]

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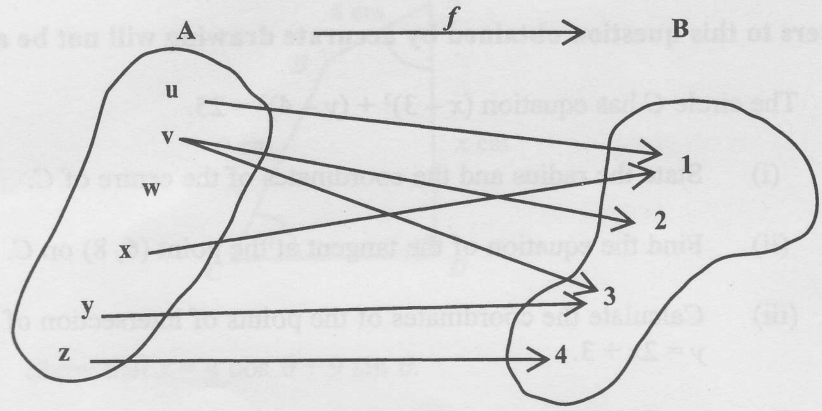
(b) The coach of an athletic club trains six athletes, u, v, w, x, y and z, in his training camp. He makes an assignment, f, of athletes u, v, x, y and z to physical activities 1, 2, 3 and 4 according to the diagram below in which $\mathbf{A} = \{u, v, w, x, y, z\}$ and $\mathbf{B} = \{1, 2, 3, 4\}$.



(i) Express f as a set of ordered pairs. **[4 marks]**

(ii) a) State TWO reasons why f is NOT a function. **[2 marks]**

b) Hence, with MINIMUM changes to f, construct a function $g : \mathbf{A} \rightarrow \mathbf{B}$ as a set of ordered pairs. **[4 marks]**

c) Determine how many different functions are possible for g in (ii) b) above. **[2 marks]**

(c) The function f on $\mathbf{R}$ is defined by

$$f(x) = \begin{cases} x - 3 & \text{if } x \leq 3 \\ \dfrac{x}{4} & \text{if } x > 3. \end{cases}$$

Find the value of

(i) $f[f(20)]$ **[3 marks]**

(ii) $f[f(8)]$ **[2 marks]**

(iii) $f[f(3)]$. **[2 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. Answers to this question obtained by accurate drawing will not be accepted.

(a) The circle C has equation $(x - 3)^2 + (y - 4)^2 = 25$.

(i) State the radius and the coordinates of the centre of C. **[2 marks]**

(ii) Find the equation of the tangent at the point (6, 8) on C. **[4 marks]**

(iii) Calculate the coordinates of the points of intersection of C with the straight line $y = 2x + 3$. **[7 marks]**

(b) The points P and Q have position vectors relative to the origin O given respectively by $\mathbf{p} = -\mathbf{i} + 6\mathbf{j}$ and $\mathbf{q} = 3\mathbf{i} + 8\mathbf{j}$.

(i) a) Calculate, in degrees, the size of the acute angle θ between $\mathbf{p}$ and $\mathbf{q}$. **[5 marks]**

b) Hence, calculate the area of triangle POQ. **[2 marks]**

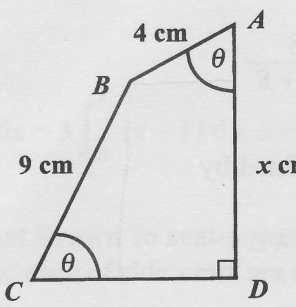
(ii) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the position vector of

a) M, where M is the midpoint of PQ **[2 marks]**

b) R, where R is such that $PQRO$, labelled clockwise, forms a parallelogram. **[3 marks]**

Total 25 marks

4. (a) The diagram below, **which is not drawn to scale**, shows a quadrilateral $ABCD$ in which $AB = 4$ cm, $BC = 9$ cm, $AD = x$ cm and $\angle BAD = \angle BCD = \theta$ and $\angle CDA$ is a right-angle.



(i) Show that $x = 4 \cos \theta + 9 \sin \theta$. **[4 marks]**

(ii) By expressing x in the form $r \cos (\theta - \alpha)$, where r is positive and $0 \leq \alpha < \frac{1}{2}\pi$, find the MAXIMUM possible value of x. **[6 marks]**

(b) Given that A and B are acute angles such that $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, find, **without using tables or calculators**, the EXACT values of

(i) $\sin (A + B)$ **[3 marks]**

(ii) $\cos (A - B)$ **[3 marks]**

(iii) $\cos 2A$. **[2 marks]**

(c) Prove that

$$\tan \left(\frac{x}{2} + \frac{\pi}{4} \right) = \sec x + \tan x.$$ **[7 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Find $\lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 6x + 8}$. **[5 marks]**

(b) The function f on **R** is defined by

$$f(x) = \begin{cases} 3 - x & \text{if } x \geq 1 \\ 1 + x & \text{if } x < 1. \end{cases}$$

(i) Sketch the graph of $f(x)$ for the domain $-1 \leq x \leq 2$. **[2 marks]**

(ii) Find

a) $\lim_{x \to 1^+} f(x)$ **[2 marks]**

b) $\lim_{x \to 1^-} f(x)$. **[2 marks]**

(iii) Deduce that $f(x)$ is continuous at $x = 1$. **[3 marks]**

(c) Differentiate from first principles, with respect to x, the function $y = \dfrac{1}{x^2}$. **[6 marks]**

(d) The function $f(x)$ is such that $f'(x) = 3x^2 + 6x + k$ where k is a constant.

Given that $f(0) = -6$ and $f(1) = -3$, find the function $f(x)$. **[5 marks]**

Total 25 marks

6. (a) Given that $y = \sin 2x + \cos 2x$, show that

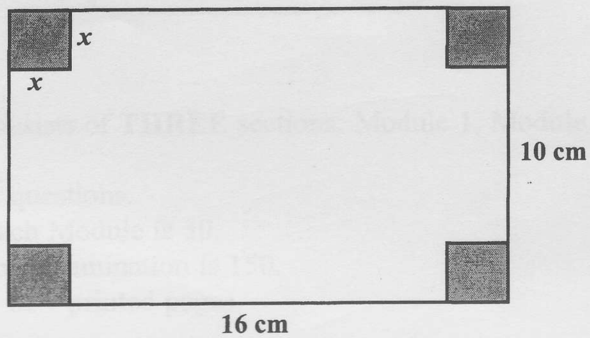
$$\frac{d^2y}{dx^2} + 4y = 0.$$

[6 marks]

(b) Given that $\int_0^a (x + 1)\,dx = 3\int_0^a (x - 1)\,dx$, $a > 0$, find the value of the constant a.

[6 marks]

(c) The diagram below **(not drawn to scale)** represents a piece of thin cardboard 16 cm by 10 cm. Shaded squares, each of side x cm, are removed from each corner. The remainder is folded to form a tray.



(i) Show that the volume, V cm^3, of the tray is given by

$$V = 4(x^3 - 13x^2 + 40x).$$

[5 marks]

(ii) Hence, find a possible value of x such that V is a maximum. **[8 marks]**

Total 25 marks

END OF TEST