FORM TP 2008241

TEST CODE **22134032**

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 03/B

ALGEBRA, GEOMETRY AND CALCULUS

1 ½ hours

26 JUNE 2008 (a.m.)

4032

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

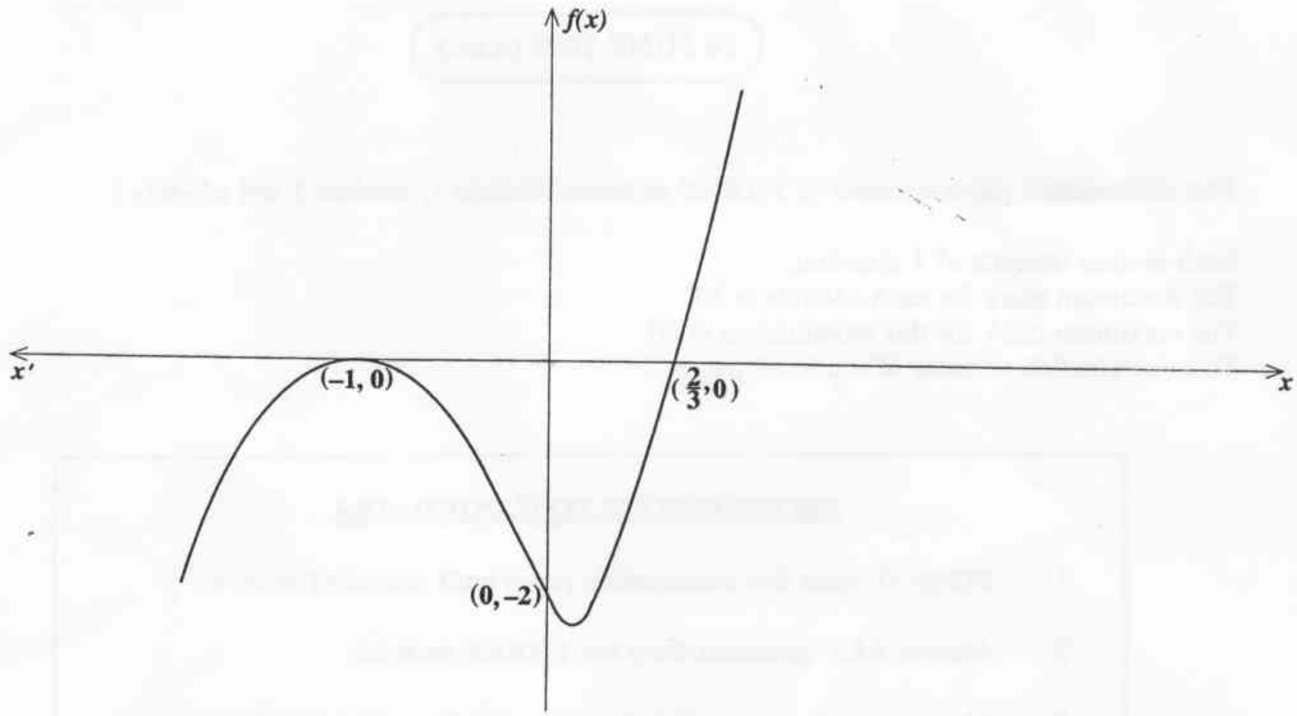
Answer this question.

1. (a) (i) Write $\log_2 2^p$ in terms of p only. **[2 marks]**

(ii) Solve for x the equation $\log_2 [\log_2 (2x - 2)] = 2$. **[3 marks]**

(b) The diagram below **(not drawn to scale)** shows the graph of the function

$f(x) = 3x^3 + hx^2 + kx + m$ which touches the x-axis at $x = -1$.



(i) Determine the values of the constants h, k and m. **[7 marks]**

(ii) State the range of values of x in $(-\infty, 0]$ for which $f(x)$ is a decreasing function. **[2 marks]**

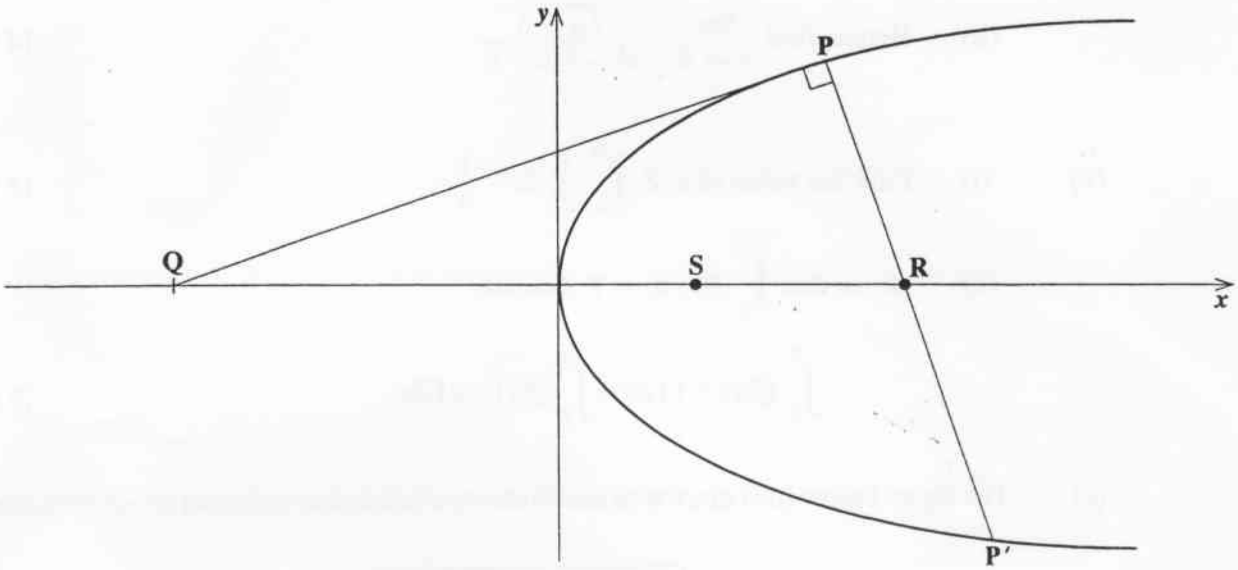
(c) Evaluate $\sum_{r=1}^{100} (3r + 2)$. **[6 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. The diagram below shows the path of a comet around the sun S. The path is described by the parametric equation $x = at^2$ and $y = 2at$, where $a > 0$ is a constant.



(a) Show that the Cartesian equation for the path is $y^2 = 4ax$. **[2 marks]**

(b) Given that the gradient m of the tangent at any point on the path satisfies $m = \dfrac{2a}{y}$,

(i) show that the equation of the tangent at (x_1, y_1) is $yy_1 = 2a\,(x + x_1)$ in Cartesian form and $ty = x + at^2$ in parametric form **[5 marks]**

(ii) find the equation of the normal at the point P with parameter t_1 **[3 marks]**

(iii) show that $t_1^{\,2} + t_1\,t_2 + 2 = 0$ if the normal in (ii) above intersects the path again at the point P' with parameter t_2 **[6 marks]**

(iv) find the distance | QR | if the tangent at P meets the x-axis at Q and the normal meets the x-axis at R. **[4 marks]**

Total 20 marks

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SECTION C (Module 3)

Answer this question.

3. (a) (i) By expressing $x - 9$ as $(\sqrt{x} + 3)(\sqrt{x} - 3)$, find $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$. **[3 marks]**

(ii) Hence, find $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x^2 - 10x + 9}$. **[4 marks]**

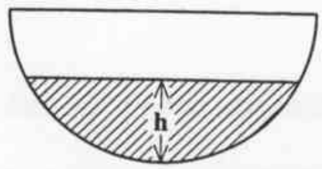
(b) (i) Find the value of u if $\displaystyle\int_u^{2u} \frac{1}{x^2}\,dx = \frac{1}{4}$. **[3 marks]**

(ii) Given that $\displaystyle\int_1^4 f(x)\,dx = 7$, evaluate

$$\int_1^2 [f(x) + 1]\,dx + \int_2^4 [f(x) - 2]\,dx.$$

[5 marks]

(c) The figure below **(not drawn to scale)** shows a hemispherical bowl which contains liquid.



The volume V cm^3 of liquid is given by

$$V = \frac{1}{3}\pi h^2 (24 - h)$$

where h is the greatest depth of the liquid in cm. Liquid is poured into the bowl at the rate of 100 cm^3 per second.

(i) Find $\dfrac{dV}{dt}$ in terms of h. **[3 marks]**

(ii) Calculate the rate at which h is increasing when $h = 2$ cm. (Leave your answer in terms of π.) **[2 marks]**

Total 20 marks

END OF TEST