FORM TP 2006258

TEST CODE **02134020**

MAY/JUNE 2006

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

2 hours

24 MAY 2006 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) Solve the simultaneous equations

$$x^2 + xy = 6$$
$$x - 3y + 1 = 0.$$

[8 marks]

(b) The roots of the equation $x^2 + 4x + 1 = 0$ are α and β. Without solving the equation,

(i) state the values of $\alpha + \beta$ and $\alpha\beta$ [2 marks]

(ii) find the value of $\alpha^2 + \beta^2$ [3 marks]

(iii) find the equation whose roots are $1 + \frac{1}{\alpha}$ and $1 + \frac{1}{\beta}$. [7 marks]

Total 20 marks

2. (a) Prove, by Mathematical Induction, that $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$. [10 marks]

(b) Express, in terms of n and in the SIMPLEST form,

(i) $\sum_{r=1}^{2n} r$ [2 marks]

(ii) $\sum_{r=n+1}^{2n} r$. [4 marks]

(c) Find n if $\sum_{r=n+1}^{2n} r = 100$. [4 marks]

Total 20 marks

Section B (Module 2)

Answer BOTH questions.

3. (a) (i) Find the coordinates of the centre and radius of the circle $x^2 + 2x + y^2 - 4y = 4$. [4 marks]

(ii) By writing $x + 1 = 3 \sin \theta$, show that the parametric equations of this circle are $x = -1 + 3 \sin \theta$, $y = 2 + 3 \cos \theta$. [5 marks]

(iii) Show that the x-coordinates of the points of intersection of this circle with the line $x + y = 1$ are $x = -1 \pm \frac{3}{2}\sqrt{2}$. [4 marks]

(b) Find the general solutions of the equation $\cos \theta = 2 \sin^2\theta - 1$. [7 marks]

Total 20 marks

4. (a) Given that $4 \sin x - \cos x = R \sin (x - \alpha)$, $R > 0$ and $0^\circ < \alpha < 90^\circ$,

(i) find the values of R and α correct to one decimal place [7 marks]

(ii) hence, find ONE value of x between 0° and 360° for which the curve $y = 4 \sin x - \cos x$ has a stationary point. [2 marks]

(b) Let $z_1 = 2 - 3i$ and $z_2 = 3 + 4i$.

(i) Find in the form $a + bi$, $a, b \in \mathbf{R}$,

a) $z_1 + z_2$ [1 mark]

b) $z_1 z_2$ [3 marks]

c) $\frac{z_1}{z_2}$. [5 marks]

(ii) Find the quadratic equation whose roots are z_1 and z_2. [2 marks]

Total 20 marks

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Section C (Module 3)

Answer BOTH questions.

5. (a) (i) State the value of $\lim_{\delta x \to 0} \frac{\sin \delta x}{\delta x}$. [1 mark]

(ii) Given that $\sin 2(x + \delta x) - \sin 2x = 2 \cos A \sin B$, find A and B in terms of x and/or δx. [2 marks]

(iii) Hence, or otherwise, differentiate with respect to x, **from first principles**, the function $y = \sin 2x$. [7 marks]

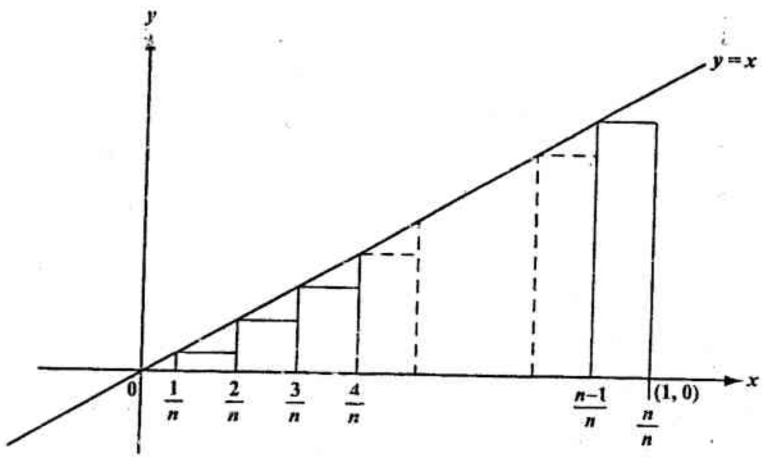
(b) The curve $y = hx^2 + \frac{k}{x}$ passes through the point P (1,1) and has a gradient of 5 at P. Find

(i) the values of the constants h and k [5 marks]

(ii) the equation of the tangent to the curve at the point where $x = \frac{1}{2}$. [5 marks]

Total 20 marks

6. (a) **In the diagram given below (not drawn to scale), the area S under the line $y = x$, for $0 \leq x \leq 1$, is divided into a set of n rectangular strips each of width $\frac{1}{n}$ units.**



(i) Show that the area S is approximately

$$\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n-1}{n^2}.$$

[6 marks]

(ii) Given that $\sum_{r=1}^{n-1} r = \frac{1}{2} n(n-1)$, show that $S \approx \frac{1}{2}(1 - \frac{1}{n})$. [2 marks]

(b) (i) Show that for $f(x) = \frac{2x}{x^2+4}$, $f'(x) = \frac{8-2x^2}{(x^2+4)^2}$. [4 marks]

(ii) Hence, evaluate $\int_0^1 \frac{24-6x^2}{(x^2+4)^2}\,dx$. [3 marks]

(c) Find the value of $u > 0$ if $\int_u^{2u} \frac{1}{x^4}\,dx = \frac{7}{192}$. [5 marks]

Total 20 marks

END OF TEST