FORM TP 2005252

TEST CODE **02134010**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 01

2 hours

20 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

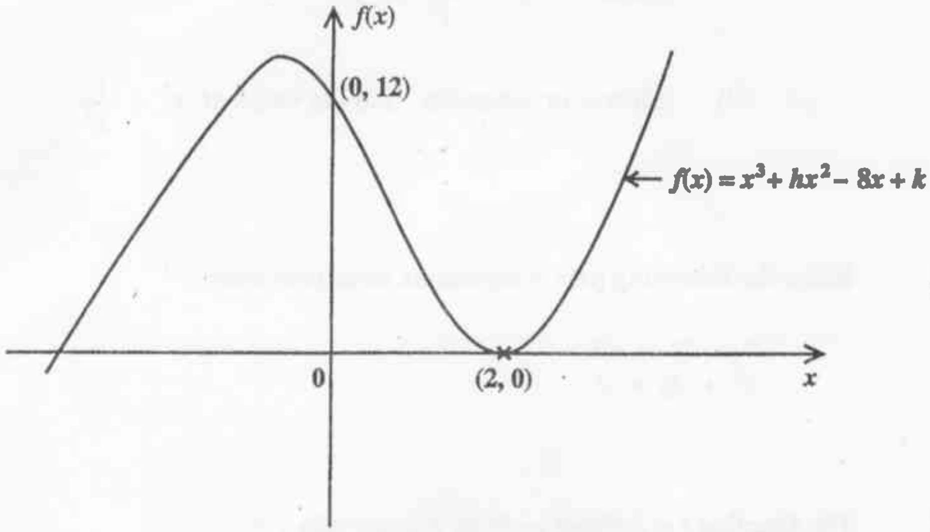
Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. The diagram below, **not drawn to scale**, shows the graph of $f(x) = x^3 + hx^2 - 8x + k$ where h, k are constants.



(a) From the graph, state the value of EACH of $f(0)$ and $f(2)$. [2 marks]

(b) Hence, or otherwise, find the value of EACH of the constants h and k. [3 marks]

(c) Factorise $f(x)$ completely. [4 marks]

Total 9 marks

2. (a) Find the range of values of the real number $x < 0$ such that

$$x^2 - 2|x| - 3 < 0.$$ [4 marks]

(b) Show that if x and y are real numbers such that $x < y$, then for any real number $k < 0$, $kx > ky$. [4 marks]

Total 8 marks

3. (a) Without using calculators or tables, show that

$$\sqrt{11} + \sqrt{7} = \frac{4}{\sqrt{11} - \sqrt{7}}.$$ [2 marks]

(b) (i) Given that $x + \frac{1}{x} = 1$, by considering $(x + \frac{1}{x})^2$

show that $x^2 + \frac{1}{x^2} = -1$. [2 marks]

(ii) Hence, or otherwise, find the value of $x^3 + \frac{1}{x^3}$. [5 marks]

Total 9 marks

4. Solve the following pair of equations simultaneously:

$$x - 2y = -3$$
$$x^2 + 3y = 7$$

Total 7 marks

5. The function f is defined on **R** by $f\colon x \rightarrow -2x + 3$.

(a) Show that f is one-to-one (injective). [2 marks]

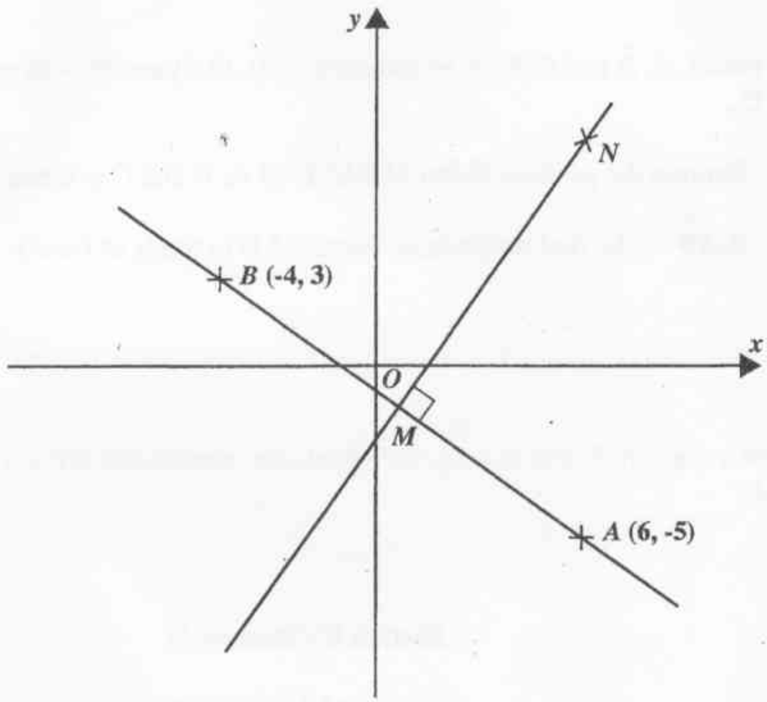
(b) Find the value(s) of $x \in \mathbf{R}$ such that $f(f(x)) = f(x) + 6$. [5 marks]

Total 7 marks

Section B (Module 2)

Answer ALL questions.

6. In the diagram below (**not drawn to scale**), M is the mid-point of AB. MN is perpendicular to the straight line through A, M and B.



(a) Find

(i) the coordinates of M **[2 marks]**

(ii) the gradient of the line through A and B **[2 marks]**

(iii) the equation of the line through M and N. **[2 marks]**

(b) The point P on AB divides AB internally such that the ratio $AP : PB$ is 3 : 1. Find the coordinates of P. **[2 marks]**

Total 8 marks

7. (a) Express $f(\theta) = \sqrt{2}\cos\theta - \sin\theta$ in the form R $\cos(\theta + \alpha)$. **[5 marks]**

(b) Hence, find the minimum value of $f(\theta)$, where $0 \leq \theta \leq 2\pi$. **[1 mark]**

(c) Determine the value of θ, $0 \leq \theta \leq 2\pi$, at which the minimum value of $f(\theta)$ occurs. **[2 marks]**

Total 8 marks

8. (a) Find the range of values of k for which the quadratic equation $x^2 + 2kx + 9 = 0$ has complex roots. **[4 marks]**

(b) Express the complex number $\dfrac{2 + 3i}{3 + 4i}$ in the form $x + yi$, where x and y are real numbers. **[4 marks]**

Total 8 marks

9. Three points, A, B and C, have coordinates (1,2), (2,5) and (0, – 4) respectively relative to the origin O.

(a) Express the position vector of EACH of A, B and C in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$. **[3 marks]**

(b) If $\overrightarrow{AB} = \overrightarrow{CD}$, find the position vector of D in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$. **[6 marks]**

Total 9 marks

10. Find the values of θ, $0 \leq \theta \leq 2\pi$, for which the vectors $\cos\theta\,\boldsymbol{i} + \sqrt{3}\,\boldsymbol{j}$ and $\frac{1}{4}\boldsymbol{i} + \sin\theta\,\boldsymbol{j}$ are parallel.

Total 7 marks

Section C (Module 3)

Answer ALL questions.

11. (a) Use the result that $(\sqrt{x+h} + \sqrt{x})(\sqrt{x+h} - \sqrt{x}) = h$ to show that

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}.$$ **[5 marks]**

(b) **Deduce**, from first principles, the derivative with respect to x of $y = \sqrt{x}$. **[1 mark]**

Total 6 marks

12. (a) Find the real values of x for which the function

$$f(x) = \frac{x}{x^2 - 2x - 8}$$

is discontinuous. **[3 marks]**

(b) Show that the equation $x^3 = 8 + 4x$ has a root in the closed interval [2, 3]. **[5 marks]**

Total 8 marks

13. P is the point on the curve $y = 2x^3 + kx - 5$ where $x = 1$ and the gradient is -2. Find

(a) the value of the constant k **[3 marks]**

(b) the value of $\frac{d^2y}{dx^2}$ at P **[2 marks]**

(c) the equation of the normal to the curve at P. **[4 marks]**

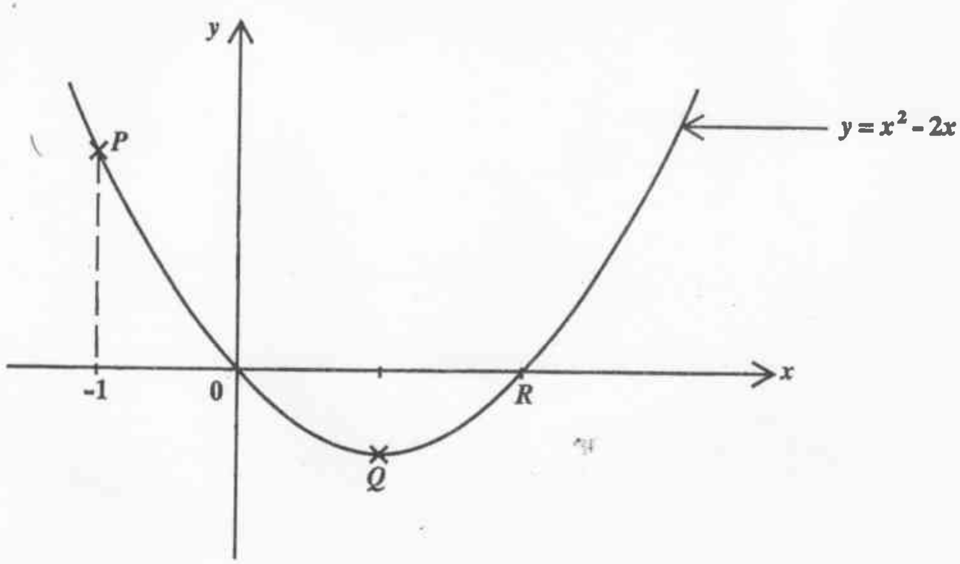
Total 9 marks

14. (a) Find the coordinates of the stationary points of the function $f: x \rightarrow x^3 - 3x^2 - 9x + 6$. **[6 marks]**

(b) Determine the nature of the stationary points of f. **[3 marks]**

Total 9 marks

15. Three points, P, Q and R, on the curve $y = x^2 - 2x$ are shown in the diagram **(not drawn to scale)** below.



(a) Find the coordinates of EACH of the points P, Q and R. **[4 marks]**

(b) Find the TOTAL area bounded by the curve shown above, the x-axis and the lines $x = -1$ and $x = 2$. **[4 marks]**

Total 8 marks

END OF TEST