TEST CODE **02134020**

FORM TP 2005253

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

2 hours

25 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) (i) Complete the table below for the function $|f(x)|$, where $f(x) = x(2-x)$.

x	-2	-1	0	1	2	3	4
$\lvert f(x) \rvert$	8				0		8

[2 marks]

(ii) Sketch the graph of $|f(x)|$ for $-2 \leq x \leq 4$. **[4 marks]**

(b) Find the value(s) of the real number, k, for which the equation $k(x^2+5) = 6 + 12x - x^2$ has equal roots. **[6 marks]**

(c) (i) If $2^{(x^2)} = 16^{(x-1)}$, find x. **[4 marks]**

(ii) Without using calculators or tables, evaluate

$(\sqrt{2}+1)^3 - (\sqrt{2}-1)^3$. **[4 marks]**

Total 20 marks

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2. (a) Prove, by Mathematical Induction, that $10^n - 1$ is divisible by 9 for all positive integers n. [9 marks]

(b) A pair of simultaneous equations is given by

$$px + 2y = 8$$
$$-4x + p^2y = 16$$

where $p \in \mathbf{R}$.

(i) Find the value of p for which the system has an infinite number of solutions. [3 marks]

(ii) Find the solutions for this value of p. [3 marks]

(c) Find the set of real values of x for which $\dfrac{x+4}{x-2} > 5$. [5 marks]

Total 20 marks

Section B (Module 2)

Answer BOTH questions.

3. The equation of the circle, Q, with centre O is $x^2 + y^2 - 2x + 2y = 23$.

(a) Express the equation of Q in the form $(x - a)^2 + (y - b)^2 = c$. [5 marks]

(b) Hence, or otherwise, state

(i) the coordinates of the centre of Q [2 marks]

(ii) the radius of Q. [1 mark]

(c) Show that the point $A(4, 3)$ lies on Q. [3 marks]

(d) Find the equation of the tangent to Q at the point A. [5 marks]

(e) The centre of Q is the midpoint of its diameter AB. Find the coordinates of B. [4 marks]

Total 20 marks

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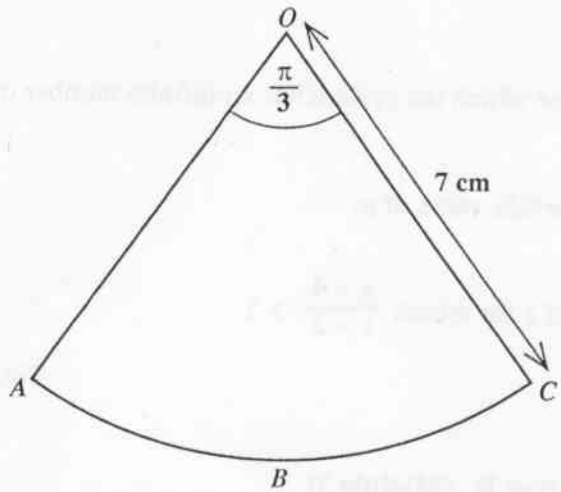
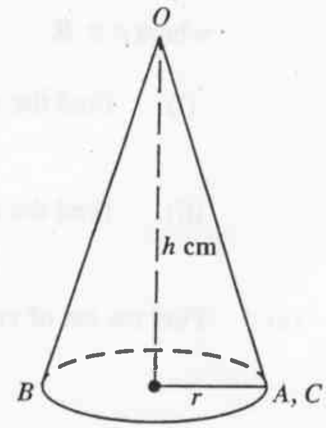
4. The diagrams shown below, **not drawn to scale**, represent

– a sector, $OABC$, of a circle with centre at O and a radius of 7 cm, where angle AOC measures $\frac{\pi}{3}$ radians.

– a right circular cone with vertex O and a circular base of radius r cm which is formed when the sector $OABC$ is folded so that OA coincides with OC.





(a) (i) Express the arc length ABC in terms of π. **[1 mark]**

(ii) Hence, show that

a) $r = \frac{7}{6}$ **[3 marks]**

b) if h cm is the height of the cone, then the exact value of h is $\frac{7\sqrt{35}}{6}$.

[2 marks]

(b) (i) Show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. **[5 marks]**

(ii) The position vectors of two points A and B relative to the origin O are

$$\boldsymbol{a} = 4\cos^2\theta\,\boldsymbol{i} + (6\cos\theta - 1)\,\boldsymbol{j}$$

$$\boldsymbol{b} = 2\cos\theta\,\boldsymbol{i} - \boldsymbol{j}.$$

By using the identity in (b) (i) above, find the value of θ, $0 \le \theta \le \frac{\pi}{4}$, such that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular. **[5 marks]**

(c) Find the modulus of the complex number $z = \frac{25(2+3i)}{4+3i}$.

[4 marks]

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

5. (a) (i) State the value of $\lim\limits_{u \to 0} \dfrac{\sin u}{u}$. [1 mark]

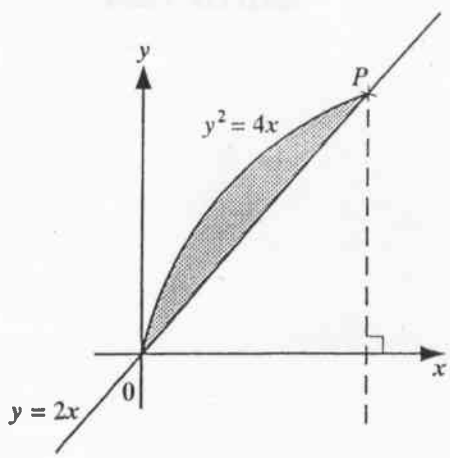
(ii) By means of the substitution $u = 3x$, show that $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x} = 3$. [4 marks]

(iii) Hence, evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 5x}$. [4 marks]

(b) If $y = \dfrac{A}{x} + Bx$, where A and B are constants, show that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y.$$ [4 marks]

(c) The diagram below, **not drawn to scale**, shows part of the curve $y^2 = 4x$. P is the point on the curve at which the line $y = 2x$ cuts the curve.



Find

(i) the coordinates of P [3 marks]

(ii) the volume of the solid generated by rotating the shaded area through 2π radians about the x-axis. [4 marks]

Total 20 marks

6. (a) Differentiate, with respect to x,

$(x^2 + 7)^5 + \sin 3x$. [6 marks]

(b) Determine the values of x for which the function $y = x^3 - 9x^2 + 15x + 4$

(i) has stationary points [3 marks]

(ii) is increasing [2 marks]

(iii) is decreasing. [2 marks]

(c) (i) Use the substitution $t = a - x$ to show that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. [4 marks]

(ii) If $\int_0^4 f(x)\,dx = 12$, use the substitution $t = x - 1$ to evaluate $\int_1^5 3f(x-1)\,dx$. [3 marks]

Total 20 marks

END OF TEST