FORM TP 02134032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

SPECIMEN PAPER

UNIT 1 – PAPER 03B 2004

$1\frac{1}{2}$ *hours*

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3

The maximum mark for each question is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL THREE** questions.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct three significant figures.

Examination Materials:

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer this question.

1. (a) Find the real values of x for which

$|2x-3| > 5.$ **[4 marks]**

(b) (i). Express $3x^2 - 4x + 1$ in the form $a(x + h)^2 + k$, stating explicitly the values of the constants a, h and k. **[3 marks]**

(ii) Hence, find the minimum value of $3x^2 - 4x + 1$ and the value of x at which the minimum occurs. **[2 marks]**

(iii) Sketch the graph of the function $3x^2 - 4x + 1$. **[4 marks]**

(c) The functions f and g are defined on **R** by

$$f : x \rightarrow x^2 - x, \quad g : x \rightarrow 2x - 3.$$

Determine the value(s) of x for which

(i) $f(x) + g(2x) = 7$ **[3 marks]**

(ii) $g^{-1}(x) = -2.$ **[4 marks]**

Total 20 marks

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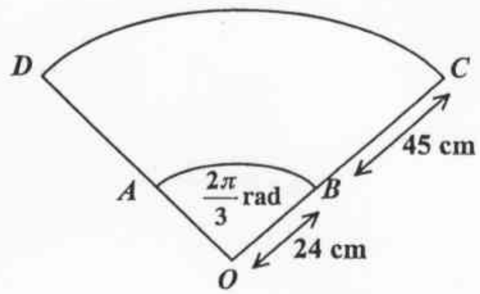
SECTION A (MODULE 2)

Answer this question.

2. (a) The diagram below **not drawn to scale,** shows the movement of a windscreen wiper of a car. It oscillates about a fixed point O and travels from OAD to OBC and back.



Given that $OB = 24$ cm, BC = 45 cm and angle DOC = $\frac{2\pi}{3}$ radians, calculate

(i) the area $ABCD$ swept by the wiper **[3 marks]**

(ii) the perimeter of $ABCD$. **[2 marks]**

(b) Prove that, for $\sin\theta \neq 1$ or -1,

(i) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} \equiv 2\sec^2\theta.$ **[3 marks]**

(ii) $\dfrac{1}{1-\sin^2\theta} - \dfrac{1}{1+\sin\theta} \equiv \tan\theta\sec\theta.$ **[3 marks]**

(c) Let x and y be non-zero real numbers, z be the complex number given by $z = x + iy$ and $\bar{z}$ be the conjugate of z. Find the values of x and y if

$$\frac{1}{z}+\frac{3}{\bar{z}} = i+1.$$ **[6 marks]**

(d) The position vectors of the points P and Q relative to the origin O are $5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{i} - 4\mathbf{j}$ respectively.

If $\overrightarrow{OP} = 3\,\overrightarrow{OQ} + 2\,\overrightarrow{OR}$, find the position vector of the point R. **[3 marks]**

Total 20 marks

SECTION C (MODULE 3)

Answer ALL questions.

3. (a) (i) Given $f(x) = x^2 + x - 3$, find $f(x + h)$. **[2 marks]**

(ii) Hence, express $\{f(x + h) - f(x)\}$ in its simplest form. **[2 marks]**

(iii) Deduce $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. **[2 marks]**

(b) Determine the real values of x for which the function

$$\frac{x}{(5x+1)^2 - 9}$$

is continuous. **[2 marks]**

(c) The curve $y = ax^2 + bx + c$ passes through the points $(0, -2)$ and $(1, -3)$ and its gradient at $x = 2$ is 5. Find

(i) the value of EACH of the constants a, b and c **[5 marks]**

(ii) the area under the curve between $x = 2$ and $x = 3$. **[3 marks]**

(d) If $\int_0^a (x - 1)\, dx = \frac{1}{2} \int_0^a (x + 1)\, dx$, and $a > 0$, find the value of a. **[4 marks]**

Total 20 marks

END OF TEST