TEST CODE **000571**

FORM TP 23240

MAY/JUNE 2003

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

1½ hours

23 MAY 2003 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination material:

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

Section A (Module 1.1)

Answer ALL questions.

1. (a) Given that $hx^3 - 12x^2 - x + 3 \equiv (2x-1)(2x+1)(x-k)$,

   find the values of the constants h and k. **[2 marks]**

   (b) Solve, for x, the equation $\dfrac{3^{(x^2)}}{27} = 9^x$. **[5 marks]**

2. Find the real values of x which satisfy the equation

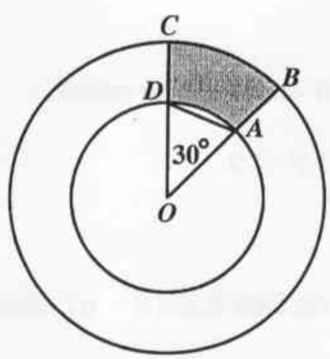
$$|2x-3|^2 - 6|2x-3| + 5 = 0.$$ **[5 marks]**

3. Given that $3 - 2x - x^2 \equiv a(x+h)^2 + k$,

   (a) state explicitly the values of the constants a, h and k **[3 marks]**

   (b) determine the maximum value of $3 - 2x - x^2$. **[2 marks]**

4. The diagram below, **not drawn to scale**, shows a circular games field of radius 35 m enclosed within a circular road of radius 42 m. The field and the road have the same centre O and angle AOD is 30°.



   (a) Find the area of the section of the road represented by the shaded region ABCD. **[4 marks]**

   (b) Find the length of the chord AD in the diagram. **[3 marks]**

5. The functions f and g are defined on **R** by

$$f: x \rightarrow 2x, \qquad g: x \rightarrow 4x + 6.$$

Find the value(s) of x such that

$$x\ g(x) = g(f(x)).$$ **[6 marks]**

Total 30 marks

Section B (Module 1.2)

Answer ALL questions.

6. (a) Find the equation of the line that is perpendicular to the line $y = 3x + 2$ and passes through the point (0, 1). **[3 marks]**

(b) (i) Find the equation of the circle with centre (1, –2) and radius 2 units. **[2 marks]**

(ii) Show, by calculation, that the line $x = 3$ touches this circle at (3, –2). **[2 marks]**

7. (a) Find the range of values of x for which

$$\frac{x}{x+1} > 0.$$ **[4 marks]**

(b) Find the range of values of x for which

$$(2x + 1)^2 \leq 9.$$ **[2 marks]**

8. Express $\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where α is acute, and hence find ALL the solutions of $\sin\theta - \cos\theta = 1$ which lie in the range $0 \leq \theta \leq \pi$. **[5 marks]**

9. (a) Express the complex number $\frac{4-2i}{1-3i}$ in the form of $a + bi$ where a and b are real numbers. **[3 marks]**

(b) Show that the argument of the complex number in (a) above is $\frac{\pi}{4}$. **[2 marks]**

10. The position vectors of two points A and B are

$-2\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{j}$ respectively.

(a) Find

(i) the unit vector in the direction of $\overrightarrow{OB}$ **[2 marks]**

(ii) the position vector of the point C on $\overrightarrow{OB}$ produced such that

$|\overrightarrow{OC}| = |\overrightarrow{OA}|$. **[2 marks]**

(b) Show that the vectors $a\mathbf{i} + b\mathbf{j}$ and $-b\mathbf{i} + a\mathbf{j}$ are perpendicular. **[3 marks]**

Total 30 marks

Section C (Module 1.3)

Answer ALL questions.

11. (a) Find $\lim_{x \to -2} \frac{x^2+x-2}{x^2+5x+6}$. **[3 marks]**

(b) Find the real values of x for which the function

$$f(x) = \frac{|x|}{(|x|^2 - 9)} \text{ is continuous.}$$ **[2 marks]**

12. Find the gradient of the tangent to the curve $y = 2x^3$ at the point where $y = 16$. **[5 marks]**

13. (a) Find the value(s) of x at the stationary point(s) of the function

$$g: x \to 2x^3 - 3x^2 + 4.$$ **[4 marks]**

(b) Determine the nature of the stationary point(s) of g. **[3 marks]**

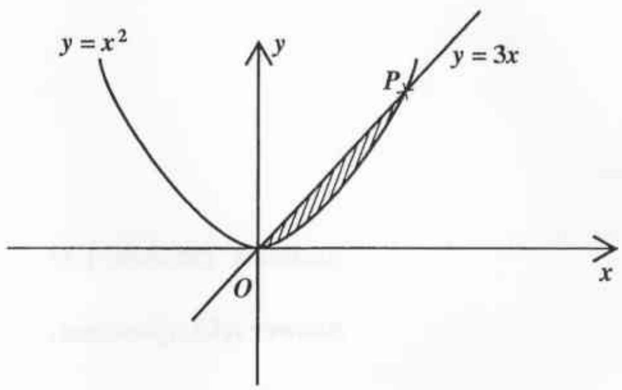
14. Find $f'(x)$ for the function

$$f(x) = \frac{x}{x^2 + 7}, \text{ and}$$

hence, or otherwise, evaluate

$$\int_{-1}^{1} \frac{14 - 2x^2}{(x^2+7)^2} \, dx.$$ **[6 marks]**

15. In the diagram below, **not drawn to scale**, the line $y = 3x$ cuts the curve $y = x^2$ at the points O and P.



Find

(a) the coordinates of the point P **[3 marks]**

(b) the area of the shaded region. **[4 marks]**

Total 30 marks

END OF TEST