TEST CODE **000572**

FORM TP 23241

MAY/JUNE 2003

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 02

2½ hours

28 MAY 2003 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

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INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination material:

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

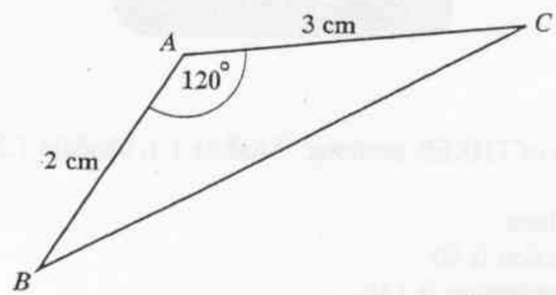
Section A (Module 1.1)

Answer BOTH questions.

1. (a) Given that $x-1$ and $x+2$ are factors of $f(x)=x^3+px+q$ where p and q are integers, find p and q.

Hence, find the remainder when x^3+px+q is divided by $(x+1)$. **[7 marks]**

(b) In the diagram shown below, **not drawn to scale**, AB = 2 cm, AC = 3 cm and $B\hat{A}C = 120°$.

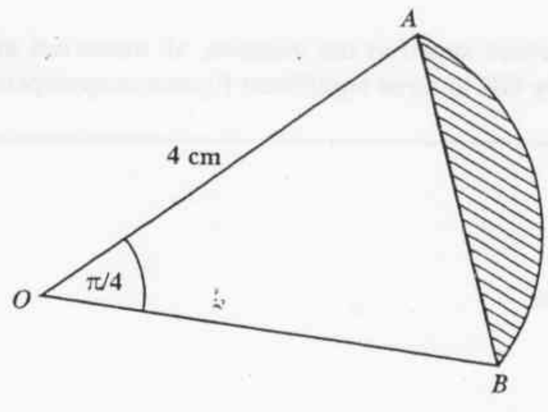


Calculate to 3 significant figures

(i) the length of BC **[4 marks]**

(ii) the value of sin C. **[4 marks]**

(c) The diagram shown below, **not drawn to scale**, is a sketch of a wedge in an electrical appliance in the form of a sector of a circle, centre O and radius 4 cm. Angle AOB measures $\frac{\pi}{4}$ radians.

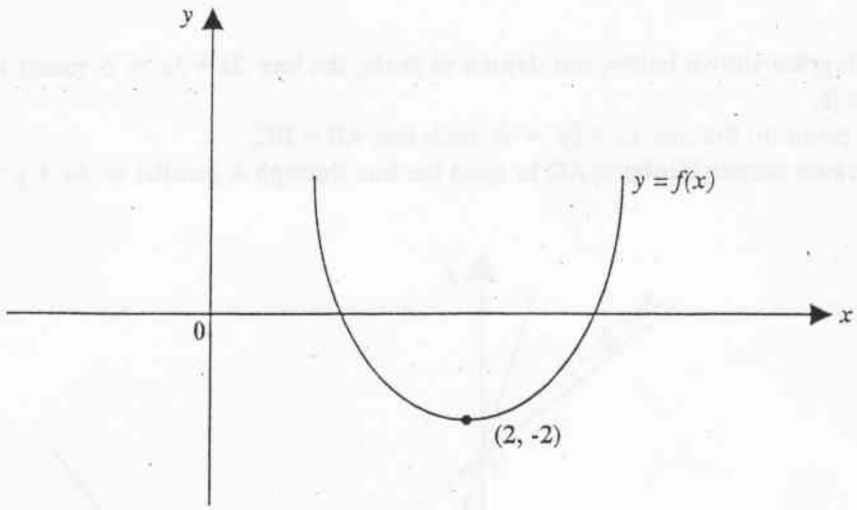


(i) Show that the area of the shaded region is $2(\pi - 2\sqrt{2})$. **[6 marks]**

(ii) Using the cosine rule, show that the length of the chord AB is $4\sqrt{(2-\sqrt{2})}$. **[4 marks]**

Total 25 marks

2. (a) The diagram below, **not drawn to scale**, shows the graph of $y = f(x)$ which has a minimum point at (2, –2).



Use this diagram to assist you in sketching the following functions:

(i) $y = f(x - 1)$ **[3 marks]**

(ii) $y = f(x) + 3$ **[3 marks]**

(iii) $y = |f(x)|$ **[3 marks]**

(b) Two sets, A and B, are defined on **R** as follows:

$$A = \{x : 0 \leq x \leq 4\}$$
$$B = \{x : 0 \leq x \leq 8\}.$$

The function $f : A \to B$ is defined by $f: x \to x(4 - x)$.

(i) Sketch the graph of $f: A \to B$. **[3 marks]**

(ii) Find a set C such that $C \subset A$ and $f: C \to B$, is one-to-one. **[3 marks]**

(iii) By considering the solutions of the equation $f(x) = 8$, show that f is NOT onto. **[4 marks]**

(iv) By solving the equation $f(x) = 0$, show that $f: A \to B$ is NOT one-to-one. **[4 marks]**

(v) Find the range of values of y for which the equation $f(x) = y$ possesses a solution. **[2 marks]**

Total 25 marks

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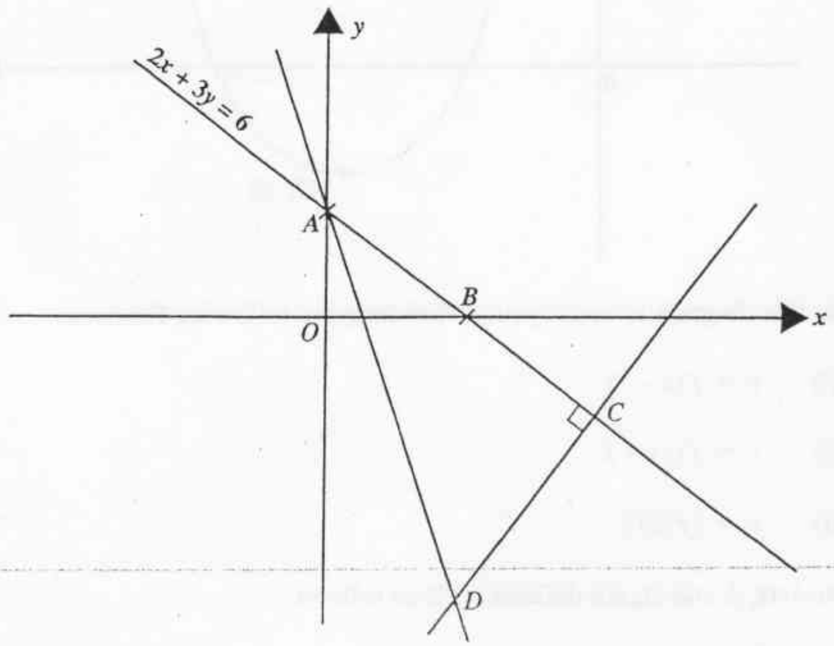
Section B (Module 1.2)

Answer BOTH questions.

3. In the diagram shown below, **not drawn to scale**, the line $2x + 3y = 6$ meets the y-axis at A and the x-axis at B.
C is the point on the line $2x + 3y = 6$ such that AB = BC.
CD is drawn perpendicular to AC to meet the line through A parallel to $5x + y = 7$ at D.



(a) Find the coordinates of A, B and C. **[7 marks]**

(b) Find the equations of the lines CD and AD. **[7 marks]**

(c) Find the coordinates of the point D. **[5 marks]**

(d) Calculate the area of triangle ACD. **[6 marks]**

Total 25 marks

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4. (a) Solve $\cos 2\theta - 3\cos\theta = 1$ for $0 \leq \theta \leq 2\pi$. **[6 marks]**

(b) If $\cos A = \frac{3}{5}$, find $\tan\frac{A}{2}$. **[6 marks]**

(c) Prove that $\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$. **[5 marks]**

(d) Given that $\sin A = \frac{12}{13}$ and $\sin B = \frac{4}{5}$, where A and B are acute angles,

find $\cos(A - B)$ and $\sin(A + B)$. **[8 marks]**

Total 25 marks

Section C (Module 1.3)

Answer BOTH questions.

5. (a) Given that $f(x) = x^3 - 5x^2 + 3x$, show that $f(x) = 0$ possesses a root in the interval $[\frac{1}{2}, 1]$.

By considering suitable values of x greater than 1,

show that there is another root of $f(x) = 0$ greater than 1. **[7 marks]**

(b) Find

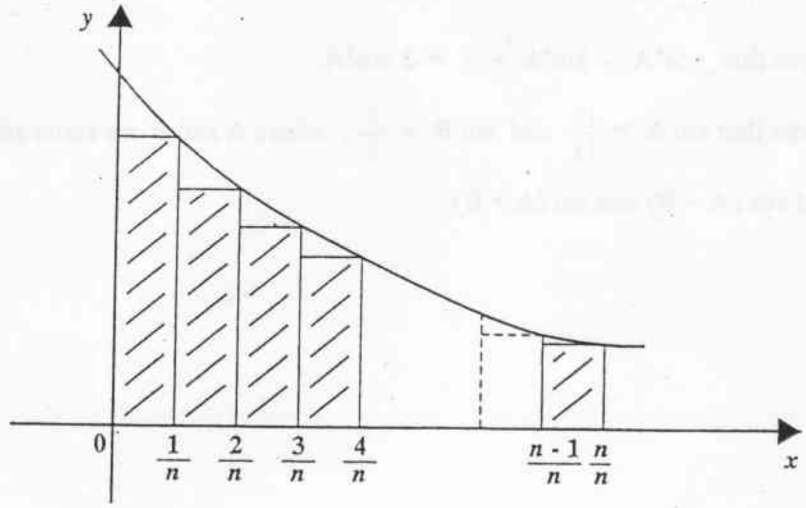
(i) the coordinates of the stationary points of $f(x)$ **[6 marks]**

(ii) the second derivative of $f(x)$, and **hence**, determine which stationary point is a local maximum and which is a local minimum. **[5 marks]**

(c) If $y = \frac{1}{x^2 + 2}$, show that $\frac{d^2y}{dx^2} = 2(3x^2 - 2)y^3$. **[7 marks]**

Total 25 marks

6. (a) In the diagram given below, **not drawn to scale**, the area under the curve $y=(1+x)^{-1}$, $0 \leq x \leq 1$, is approximated by a set of n rectangular strips each of width $\frac{1}{n}$ units.



Show that the sum, S_n, of the areas of the rectangular strips is $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}$.

[7 marks]

(b) (i) Show that for $f(x) = \frac{x}{x^2+4}$,

$$f'(x) = \frac{4-x^2}{(x^2+4)^2}.$$

[4 marks]

(ii) **Hence**, evaluate

$$\int_0^2 \frac{12-3x^2}{(x^2+4)^2}\, dx.$$

[4 marks]

(c) (i) Sketch the curve $y = x^2 + 1$. **[3 marks]**

(ii) Find the volume obtained by rotating the portion of the curve between $x = 0$ and $x = 1$ through 2π radians **about the** y **axis**. **[7 marks]**

Total 25 marks

END OF TEST