FORM TP 22241

TEST CODE **000571**

MAY/JUNE 2002

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

$1\frac{1}{2}$ hours

24 MAY 2002 (a.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1.1)

Answer ALL questions.

1. If $x = 2$ is a root of the equation $6x^3 - px^2 - 14x + 24 = 0$, find p.

   Hence, find the other roots of the equation. **[6 marks]**

2. Solve for x the equation $2^{2x} - 3.2^{x+1} + 8 = 0$. **[4 marks]**

3. The function, f, is defined on $\mathbf{R}$ by

   $$f: x \to x^2 - 3.$$

   Determine the set of values of $x \in \mathbf{R}$ for which

   $$f[f(x)] = f(x + 3).$$

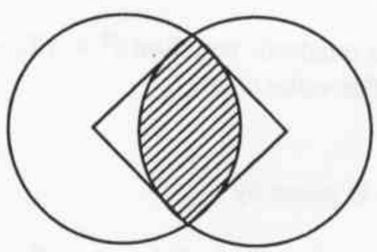
   [7 marks]

4. Solve the simultaneous equations

   $$2x - y = 5$$
   $$x^2 - 6y = xy.$$

   [7 marks]

5. The shape of the earring pendant, shown in the shaded portion of the diagram below, is obtained from two equal overlapping circles. The height of the pendant is 2 cm and is equal to the radius of the circles. Find the area of the pendant.



[6 marks]

Total 30 marks

Section B (Module 1.2)

Answer ALL questions.

6. A, B and C are three points on a straight line, and B is the mid-point of AC. The coordinates of A and B are $(-1, 1)$ and $(2, 0)$ respectively. Find

(a) the coordinates of C **[3 marks]**

(b) the equation of the straight line through the origin, O, perpendicular to OC. **[3 marks]**

7. A pair of simultaneous equations is given by

$$p^2x - 4y = 8$$
$$8x - 2y = p$$

where $p \in \mathbf{R}$.

(a) Find the value of p for which the equations have an infinite number of solutions. **[4 marks]**

(b) Find the solutions for this value of p. **[2 marks]**

8. Prove that $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$. **[5 marks]**

9. (a) The roots of the quadratic equation $x^2 - 3x - 1 = 0$ are α, β. Without solving the equation, obtain the equation whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$. **[4 marks]**

(b) One root of the quadratic equation $x^2 + 12x - a = 0, a \in \mathbf{R}$, is three times the other. Find the roots and the value of a. **[3 marks]**

10. (a) The vector, $\mathbf{r}$, is given by

$$\mathbf{r} = (\cos\theta + 2\sin\theta)\,\mathbf{i} + (\sin\theta - 2\cos\theta)\,\mathbf{j}.$$

Show that the modulus of $\mathbf{r}$ is independent of θ. **[3 marks]**

(b) Find the vector parallel to $\mathbf{i} + 3\mathbf{j}$ which has the same magnitude as $2\mathbf{i} - \mathbf{j}$. **[3 marks]**

Total 30 marks

Section C (Module 1.3)

Answer ALL questions.

11. (a) Find $\lim_{x \to -2} \dfrac{x + 2}{2x^3 - 8x}$. **[4 marks]**

(b) Determine the real values of x for which the function

$$f(x) = \frac{2x + 1}{x^2 + x - 2}$$

is continuous. **[2 marks]**

12. Find $f'(x)$ for the function

$$f(x) = \frac{x^2}{x^3 + 2}.$$

Hence, or otherwise, evaluate

$$\int_0^1 \frac{16x - 4x^4}{(x^3 + 2)^2}\, dx.$$ **[6 marks]**

13. (a) Find the stationary point(s) of the function

$f : x \to 27x - x^3$. **[3 marks]**

(b) Determine the nature of the stationary point(s) of f. **[3 marks]**

14. Use the substitution $u = \sin x$ to find

$$\int \cos^3 x\, dx.$$ **[6 marks]**

15. (a) Draw a rough sketch of the curve $y = x^2 + x$. **[2 marks]**

(b) Find the total area bounded by the curve in Part (a) above, the x-axis and the lines $x = -1$ and $x = 3$. **[4 marks]**

Total 30 marks

END OF TEST