FORM TP 22242

TEST CODE **000572**

MAY/JUNE 2002

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 02

$2\frac{1}{2}$ *hours*

29 MAY 2002 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1.1)

Answer BOTH questions.

1. Let A $= \{x \in \mathbf{R}: x > 2\}$ and B $= \{x \in \mathbf{R}: x > 0\}$. Let f : A $\rightarrow$ B be the function given by

$$f : x \rightarrow \frac{2x}{x-2}.$$

(a) Find $p, q \in$ R such that $f(x) = p + \frac{q}{x-2}$. **[3 marks]**

(b) Show that f is one-to-one. **[6 marks]**

(c) Determine whether there is an $x \in$ A such that $f(x) = 1$. **[7 marks]**

(d) Use Part (c) above to determine

(i) the range of f **[5 marks]**

(ii) whether or not f is onto. **[4 marks]**

Total 25 marks

2. (a) If $t = \tan {}^1\!/_2\, \theta$, express $\cos \theta$ and $\sin \theta$ in terms of t. **[6 marks]**

Hence, find $\tan {}^1\!/_2\, \theta$ when $\cos \theta + 2 \sin \theta = {}^{11}\!/_5$. **[7 marks]**

(b) In triangle ABC, $A\hat{C}B = 90°$ and D is the point on BC such that $A\hat{B}D = B\hat{A}D = \theta°$. Given that BD = 5 cm and AB = 8 cm, find

(i) $\cos \theta$ **[3 marks]**

(ii) $\sin \theta$ **[3 marks]**

(iii) the length of BC **[3 marks]**

(iv) the length of AC. **[3 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

Section B (Module 1.2)

Answer BOTH questions.

3. (a) A curve is given by the parametric equations

$$x = 4t^2, \; y = 8t.$$

(i) Find $\frac{dy}{dx}$ in terms of t. [4 marks]

(ii) Show that the Cartesian equation of the tangent to the curve at the point P with parameter T is

$$Ty = x + 4T^2.$$ [5 marks]

(iii) The tangent in (ii) above meets the y-axis at the point Q and the x-axis at the point R. If O is the origin, show that the area of triangle OQR is $8T^3$ square units. [6 marks]

(b) Solve for $x \in \mathbf{R}$ the inequality

$$\frac{2x + 3}{3x + 4} < 1.$$ [10 marks]

Total 25 marks

4. (a) Find the two square roots of the complex number $5 - 12i$ in the form $x + yi$, where $x, y \in \mathbf{R}$. [8 marks]

(b) (i) If $z = x + yi$, where $x, y \in \mathbf{R}$, $y \neq 0$, find the real and imaginary parts of $z + \frac{1}{z}$. [5 marks]

(ii) Find and identify the locus of the points for which the imaginary part of $z + \frac{1}{z}$ is zero. [5 marks]

(c) If the position vector of the point A is $\mathbf{i} - 3\mathbf{j}$ and the position vector of the point B is $2\mathbf{i} + 5\mathbf{j}$, find

(i) $\left|\overrightarrow{AB}\right|$ [4 marks]

(ii) the position vector of the mid-point of $\overrightarrow{AB}$. [3 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

Section C (Module 1.3)

Answer BOTH questions.

5. (a) By expressing $x - 4$ as $(\sqrt{x} + 2)(\sqrt{x} - 2)$, find

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}.$$

Hence, find $\displaystyle\lim_{x \to 4} \frac{\sqrt{x} - 2}{x^2 - 5x + 4}$. [5 marks]

(b) The equation of a curve is given by

$$f(x) = x(x + 2)^2.$$

(i) Obtain an expression for $f'(x)$. [3 marks]

(ii) Find the stationary point(s) of f. [2 marks]

(iii) Determine the nature of the stationary point(s) of f. [5 marks]

(iv) Sketch the curve. [5 marks]

(v) Find the area bounded by the curve and the interval of the x-axis, $-2 < x < 0$. [5 marks]

Total 25 marks

6. (a) Using the substitution $u = x + 3$ or otherwise, evaluate

$$\int x\sqrt{x + 3}\ dx.$$ [6 marks]

(b) The section of the curve $y = x\sqrt{x + 3}$, $0 \le x \le 2$, is rotated about the x-axis through 360°. Find the volume generated by

(i) direct integration [8 marks]

(ii) the trapezium rule, using five coordinates. [11 marks]

Total 25 marks

END OF TEST