FORM TP 21222

TEST CODE **000571**

MAY/JUNE 2001

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

$1\frac{1}{2}$ hours

25 MAY 2001 (a.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

SECTION A (MODULE 1.1)

Answer ALL questions.

1. Solve for x, $|2x-1|-3 = x$, $x \in \mathbf{R}$. **[5 marks]**

2. Find real numbers a and b such that

$$x^2 + 5x - 1/4 = (x + a)^2 + b.$$

[5 marks]

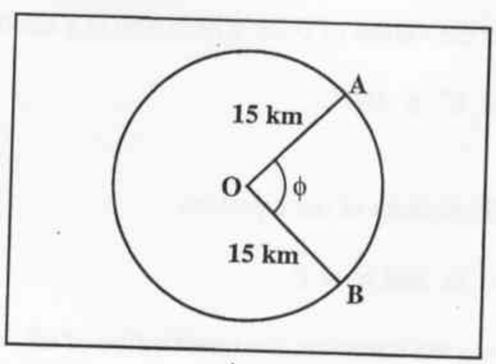
3. Solve the equation, $\dfrac{4^{2x}}{(1/2)^{x^2}} = 32.$ **[6 marks]**

4. Solve the pair of simultaneous equations

$$y^2 - xy = 2$$
$$2x + y = 1.$$

[7 marks]

5.



The diagram above, not drawn to scale, shows part of the radar system of a coast guard boat. O represents the position of the coast guard station. A represents the position of the coast guard boat at sea and B represents the position of a fishing vessel from which emanates a distress signal. OA = 15 km and $\angle AOB = \phi°$.

(a) Find the expression, in terms of π and ϕ, for

(i) the length of the minor arc, AB **[2 marks]**

(ii) the area of the corresponding sector, OAB. **[2 marks]**

(b) Given that $\phi = 40$, calculate the area of the triangular region, mapped out between the coast guard station, the boat and the vessel. **[3 marks]**

Total 30 marks

SECTION B (MODULE 1.2)

Answer ALL questions.

6. Given the points, A(3, –5) and B (–4, 2), find

(i) the coordinates of the mid-point M of AB **[1 mark]**

(ii) the gradient of AB **[1 mark]**

(iii) the equation of the straight line through M that is normal to AB. **[3 marks]**

7. Sketch the graph of the function

$$f(x) = x(x + 4) - 5.$$

Hence, solve the inequality, $x(x + 4) > 5$. **[6 marks]**

8. (a) Express $f(\theta) = \sqrt{2}\cos\theta + \sin\theta$ in the form $R\cos(\theta + \alpha)$. **[5 marks]**

(b) Hence, find the values of θ for which $f(\theta)$ is a maximum or a minimum, where $0° \leq \theta° \leq 360°$. **[2 marks]**

9. Given that α and β are roots of the equation

$$4 - 2x^2 = 3x \text{ and } \alpha > \beta,$$

find, without solving the equation, the exact value of $\alpha^2 - \beta^2$. **[6 marks]**

10. Given that $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{CB} = 5\mathbf{i} + \mathbf{j}$

(a) show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$ **[3 marks]**

(b) find the unit vector in the direction of $\overrightarrow{AB}$. **[3 marks]**

Total 30 marks

SECTION C (MODULE (1.3)

Answer ALL questions.

11. Given that $\lim_{x \to 3} \{f(x) + 3x\} = 1$,

evaluate $\lim_{x \to 3} \{9f(x)\}$. **[5 marks]**

12. Show that the equation, $x^3 = 10 - 3x$, has a root between 1 and 2. **[5 marks]**

13. The curve, $y = ax^2 + bx$, where a and b are real constants, has a stationary point at (1,2). Calculate the respective values of a and b. **[6 marks]**

14. Differentiate $\dfrac{x^2 + 2}{2x^2 - 1}$ with respect to x. **[3 marks]**

Hence, or otherwise, find $\displaystyle\int \frac{50x}{(2x^2 - 1)^2}\,dx$. **[3 marks]**

15. Initially, the depth of water in a tank is 32 m. Water drains from the tank through a hole cut in the bottom. At t minutes after the water begins draining, the depth of water in the tank is x metres. The water level changes at a rate equal to $(-2t - 4)$.

(a) Find an expression for x in terms of t. **[5 marks]**

(b) Hence, determine how long it takes for the water to completely drain from the tank. **[3 marks]**

Total 30 marks

END OF TEST