FORM TP 21223

TEST CODE **000572**

MAY/JUNE 2001

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 - PAPER 02

$2\frac{1}{2}$ *hours*

12 JUNE 2001 (p.m.)

21223

This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 7 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

SECTION A (MODULE 1.1)

Answer BOTH questions.

1. (a) Show that the inequality

$$|x + 3| \geq |x| - |3|$$

holds for any $x \in \mathbf{R}$. **[4 marks]**

(b) Factorise completely the polynomial, $x^3 + 2x^2 - x - 2$. **[4 marks]**

(c) Find the value of r such that $x^4 - 7x + 4r$ has a remainder -2 when it is divided by $x + 3$. **[4 marks]**

(d) The functions, f and g, are defined by

$$f: x \to \frac{x^2}{1 - x^2},\ x^2 \neq 1, \text{ and } g : x \to \frac{1}{2}x - 3.$$

(i) Explain clearly why f is not $1 - 1$. **[3 marks]**

(ii) Calculate and simplify $g\,f(x)$. **[5 marks]**

(e)

Time, t, minutes	0	10	20	30	40	50
Numbers of Bacteria, n	5	10	20	40	80	160

The table above shows the growth in bacterial numbers, n, with time, t, in minutes, as recorded during a laboratory experiment. After treatment with an antibiotic, the number of bacteria is reduced at the same rate for the next 50 minutes.

(i) Using a scale of 2 cm to represent 10 minutes on the x-axis and 2 cm to represent 20 bacteria on the y-axis, draw the growth curve for the treatment stage. **[4 marks]**

(ii) Hence, estimate the number of bacteria present after the first 35 minutes of treatment. **[1 mark]**

Total 25 marks

2. (a) The function, f, is given by $f(x) = 2x^2 + 11x + 3$.

(i) Express $f(x)$ in the form $p(x - q)^2 + r$, where $p, q, r \in \mathrm{R}$. **[3 marks]**

(ii) Sketch the graph of $f(x)$. **[2 marks]**

(iii) Hence, or otherwise, find the turning point of the graph and determine whether it is a maximum or a minimum. **[3 marks]**

(b) Given that δ is an acute angle such that $\cos \delta = \frac{1}{4}x$,

find the expression for $\sin 2\delta$ in terms of x. **[4 marks]**

(c) The parametric equations of a curve, C, are

$$2x = \cot \theta - \operatorname{cosec} \theta \text{ and } 4y = \operatorname{cosec} \theta - 2 \cot \theta.$$

(i) Express $\cot \theta$ and $\operatorname{cosec} \theta$ in terms of x and y. **[3 marks]**

(ii) Find an equation connecting x and y. **[2 marks]**

(d) (i) Sketch, in separate diagrams, the graphs of $y = \sin x$ and $y = \cos x$ for $-2\pi \leq x \leq 2\pi$. **[6 marks]**

(ii) State clearly the transformation which maps $y = \cos x$ onto $y = \sin x$. **[2 marks]**

Total 25 marks

SECTION B (MODULE 1.2)

Answer BOTH questions.

3. (a) A pair of simultaneous equations is given by

$2x + py = 13$
$px + 32y = 52,$

where $p \in \mathbf{R}$.

Find the value(s) of p for which the system of equations above has

(i) a unique solution **[3 marks]**

(ii) an infinite number of solutions **[3 marks]**

(iii) no solution. **[3 marks]**

(b) Solve the following equations for $0 \leq x \leq \pi/_2$:

(i) $6 \sin^2 x - \cos x - 4 = 0$ **[5 marks]**

(ii) $\sqrt{3}\ \sin x + \cos x = 2$ **[5 marks]**

(c) Show that, for $A \neq \pi/_2$, $\sec A - \tan A = \tan\left(\frac{\pi}{4} - \frac{A}{2}\right)$. **[6 marks]**

Total 25 marks

4. (a) The complex number, z, is expressed in the form $x + iy$, where $x, y \in \mathbf{R}$.

Express the complex number, $\frac{z-1}{z+1}$, in a similar form. **[7 marks]**

The argument of the complex number, $\frac{z-1}{z+1}$, is $\frac{\pi}{4}$.

(i) Find the equation connecting x and y. **[3 marks]**

(ii) Show that the equation represents a circle, C. **[3 marks]**

(iii) Determine the centre and radius of C. **[2 marks]**

(b) (i) $\mathbf{r}_1 = 3\mathbf{i}$, $\mathbf{r}_2 = \mathbf{i} + \mathbf{j}$ and $\mathbf{r}_3 = -4\mathbf{j}$ are three vectors and t is a scalar.

Find the values of t such that the vector $t\,\mathbf{r}_1 + \mathbf{r}_2$ is perpendicular to the vector $t\,\mathbf{r}_2 + \mathbf{r}_3$.

[5 marks]

(ii) The position vectors of points A and B relative to the origin, O, are $4\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + 7\mathbf{j}$ respectively. The point, C, lies on AB such that AC : CB = 2 : 1. Find the position vector of C relative to O. **[3 marks]**

(iii) Determine $\cos A\hat{O}C$. **[2 marks]**

Total 25 marks

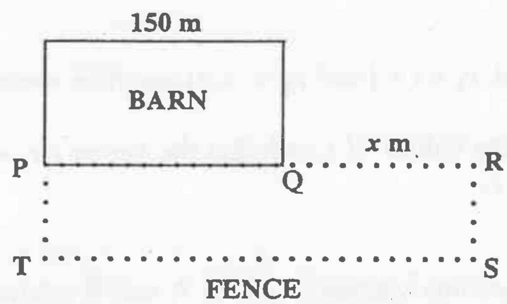
SECTION C (MODULE 1.3)

Answer BOTH questions.

5. (a) Using differentiation, determine the range of real values of x for which the function $f : x \rightarrow 12 + 6x^2 - x^3$ is decreasing. **[5 marks]**

(b) Differentiate x^3 with respect to x from first principles. **[6 marks]**

(c) A farmer plans to construct an enclosure for his sheep, making use of one side of a barn that is 150 m in length. Using 500 m of fencing material, the farmer will build a fence QRSTP which, along with the existing barn wall, PQ, will form a rectangular enclosure PRST.



Let the length of QR be x metres. Simplifying your answers where possible, find expressions in terms of x for

(i) the length of TS **[1 mark]**

(ii) the length of RS **[4 marks]**

(iii) the area, A, of PRST **[2 marks]**

(iv) the stationary value of x and show that it is a maximum **[5 marks]**

(v) the maximum area of the sheep enclosure. **[2 marks]**

Total 25 marks

6. (a) Calculate the volume generated when the finite region in the first quadrant bounded by the curve, $y = 2x^2$, the y-axis and the line $y = 2$ is rotated completely about the y-axis. **[6 marks]**

(b) The region, R, is bounded by the curve $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 2$.

(i) Calculate the area of R. **[5 marks]**

(ii) The area of R is estimated using the trapezium rule with 2 intervals of equal width. Show that this trapezium rule estimate differs by $\frac{1}{3}$ from the exact value for the area of R found in (b)(i). **[5 marks]**

(iii) On a carefully labelled sketch of $y = x^2 + 1$, shade in the 2 trapezia which are used to estimate the area of R. **[4 marks]**

(iv) Another approximation for the area of R is obtained using 2 trapezia of unequal width. The first trapezium has width, h, and the second trapezium width, $(2 - h)$ with the three ordinates occurring where $x = 0$, $x = h$ and $x = 2$.
Show that the total area of these 2 trapezia of unequal widths is given by $(h^2 - 2h + 6)$. **[5 marks]**

Total 25 marks

END OF TEST