TEST CODE **000571**

FORM TP 20217

MAY 2000

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

$1\frac{1}{2}$ hours

24 MAY 2000 (a.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

SECTION 1 (MODULE 1.1)

Answer ALL questions.

1. (a) If $x^3 + 2x^2 - 5x - 6 = (x-2)(x+3)(x-k)$, find the value of k. **[2 marks]**

   (b) Find the real values of x which satisfy the equation

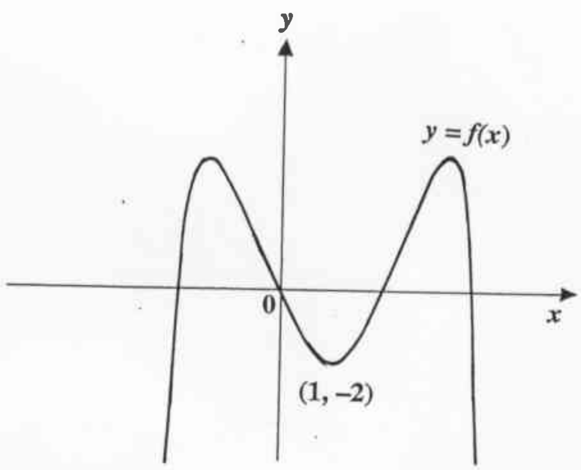
$$|2x + 3| = 5.$$

[3 marks]

2. (a) Express $1 - 2x - 3x^2$ in the form $a(x+h)^2 + k$, stating explicitly the values of a, h and k. **[3 marks]**

   (b) Solve for x, the equation $2^{3-5x} = \left(\frac{1}{64}\right)^{x-1}$. **[3 marks]**

3. The diagram below shows the graph $y = f(x)$ which has a minimum point at $(1, -2)$.



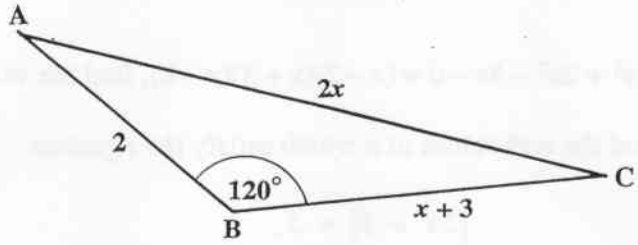
State the coordinates of the minimum point on the graph with equation

(a) $y = f(x) + 2$ **[2 marks]**

(b) $y = f(x - 3)$ **[2 marks]**

(c) $y = 4f(x)$. **[2 marks]**

4. In triangle ABC, $\angle ABC = 120^\circ$, AB = 2 *m*, AC = 2*x* *m* and BC = $(x + 3)$ *m*.



Show that $3x^2 - 8x - 19 = 0$. **[6 marks]**

5. The functions *f* and *g* are defined on **R** by

$f: x \rightarrow 2x$
$g: x \rightarrow x^2 - 3.$

Determine the set of values of *x* for which $f(f(x)) = g(f(x))$. **[7 marks]**

Total 30 marks

SECTION 2 (MODULE 1.2)

Answer ALL questions.

6. Let α, β be the roots of the equation

$$4x^2 - 3x + 1 = 0.$$

(a) Without solving the equation, write down the values of

(i) $\alpha + \beta$

(ii) $\alpha\beta$. **[1 mark]**

(b) Find the value of $\alpha^2 + \beta^2$. **[2 marks]**

(c) Find the equation whose roots are

$$\frac{2}{\alpha^2} \text{ and } \frac{2}{\beta^2}.$$ **[3 marks]**

7. Prove that $\dfrac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$. **[5 marks]**

8. (a) Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta+\alpha)$, where R is a positive real constant and $0^\circ < \alpha < 90^\circ$. **[4 marks]**

(b) Hence, deduce the minimum value of $\sin\theta + \sqrt{3}\cos\theta + 6$.

[3 marks]

9. Let A (1, 2) be a point in the coordinate plane with origin O. Find

(a) the equation of the straight line, OA **[3 marks]**

(b) the equation of the straight line, AB, through A perpendicular to OA **[2 marks]**

(c) the coordinates of the point B, at, which the line AB crosses the x-axis.

[1 mark]

10. (a) In a triangle ABC, the position vectors of A, B and C are respectively

$$\mathbf{i} + \mathbf{j},\ 3\mathbf{i} + 4\mathbf{j} \text{ and } 4\mathbf{i} - \mathbf{j}.$$

(i) Find $\overrightarrow{BA}$ and $\overrightarrow{AC}$. **[2 marks]**

(ii) Show that $\angle BAC = 90^\circ$. **[2 marks]**

(b) A pair of simultaneous equations is given by

$$px + 4y = 8$$
$$6x + 2y = q$$

where $p, q \in \mathbf{R}$.

State the values of p and q for which the simultaneous equations have an infinite number of solutions. **[2 marks]**

Total 30 marks

SECTION 3 (MODULE (1.3)

Answer ALL questions.

11. (a) Find $\lim\limits_{x\to 3} \dfrac{x^2-9}{3x^2-9x}$. **[3 marks]**

(b) Determine the real values of x for which the function

$$f(x) = \frac{x}{|2x|-7}$$

is continuous. **[3 marks]**

12. (a) Find the value of x at the stationary point(s) of the function

$f: x \to 4x - 3x^3$. **[3 marks]**

(b) Determine the nature of the stationary point(s). **[3 marks]**

13. The function, f, is defined by

$$f(x) = \frac{(x^6+x^4-x^2)}{x^4},\ x\in\mathrm{R},\ x\neq 0.$$

(a) Find $\int (f(x)+3)\,dx$. **[3 marks]**

(b) Evaluate $\int_1^3 (f(x)+3)\,dx$. **[3 marks]**

14. (a) Given that $\int_0^2 f(x)\,dx=17$ and $\int_0^4 f(x)\,dx=22$, where $f(x)$ is a real continuous function in the closed interval [0, 4], evaluate

$\int_2^4 f(x)\,dx$. **[2 marks]**

(b) Differentiate with respect to x, from first principles, the function

$y = x^2 + 2$. **[4 marks]**

15. Let f be a cubic function in x. Suppose that $f(x) = 0$ has roots at $x = 0$ and $x = 3$, and f has a maximum point at (1, 4) and a minimum point at (3, 0). Sketch the graph of $y = f(x)$ indicating clearly its maximum point, its minimum point and its intercepts with the axes.

[6 marks]

Total 30 marks

END OF TEST