TEST CODE **000572**

FORM TP 20218

MAY 2000

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 02

$2\frac{1}{2}$ *hours*

13 JUNE 2000 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination paper consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

20218

Examination materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

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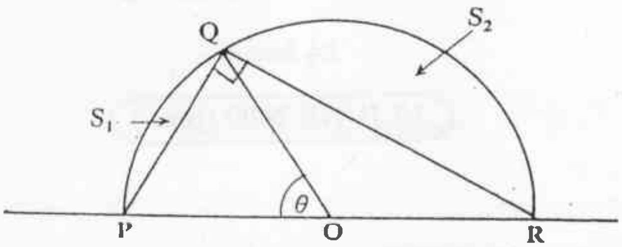
SECTION I (MODULE 1.1)

Answer BOTH questions.

1. (a) The straight line $y = 10 - 3x$ cuts the circle $x^2 + y^2 + 2x - 16y = 0$ at the points, A and B. Given that A lies in the first quadrant and B lies in the second quadrant, find the coordinates of A and B. **[9 marks]**

(b)



The diagram above, not drawn to scale, shows a semicircle, PQR, with PR as diameter, O as centre and radius, r cm. The angle POQ = θ radians, $0 < \theta < \pi/2$.

S_1 is the segment of the circle bounded by the chord, PQ, and S_2 is the segment of the circle bounded by the chord, QR.

Write down, in terms of θ and r, an expression for

(i) the area of S_1 **[3 marks]**

(ii) the area of S_2 . **[4 marks]**

(iii) Given that the area of S_2 is three times the area of S_1, show that

$$4\theta = \pi + 2 \sin \theta.$$ **[4 marks]**

(iv) Show that when $r = 6$, $\theta = \frac{\pi}{3}$, the area of triangle PQR = $18\sqrt{3}$. **[5 marks]**

Total 25 marks

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2. (a) The function, f, is defined by

$$f: x \rightarrow x^2 - 2x + 3, x \in \mathrm{R}, x \geq 1.$$

(i) State the range of f. [3 marks]

(ii) Sketch the graph of f. [3 marks]

(iii) Explain why the inverse function f^{-1} of f exists. [3 marks]

(iv) State the domain of f^{-1}. [2 marks]

(v) Sketch f^{-1} on the same diagram as f. [4 marks]

(b) The function, g, is defined by

$$g: x \rightarrow x^2, x \in \mathrm{R}, x \geq 0.$$

State clearly the transformation which maps g onto f. [4 marks]

(c) The function, h, is given by

$$h: x \rightarrow \frac{1}{x}, x \in \mathrm{R}, x \neq 0.$$

(i) Find an expression for $f(h(x))$. [3 marks]

(ii) Write down the range for the function $f(h(x))$. [3 marks]

Total 25 marks

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SECTION 2 (MODULE 1.2)

Answer BOTH questions.

3. (a) (i) Express the complex number $z = \dfrac{11-2i}{3+4i}$ in the form $a + ib$, where a and b are real numbers. **[4 marks]**

(ii) Hence, express z^2 and iz in a similar form. **[3 marks]**

(iii) Find the modulus and principal value of the argument of z, where $-\pi < \arg z \leq \pi$. **[3 marks]**

(iv) Find the exact distance between the points on the Argand diagram represented by z^2 and iz. **[3 marks]**

(b) (i) Find the cartesian equation of the curve, C, with parametric equations $x = 3 + 5 \sin \theta$, $y = 4 + 5 \cos \theta$, where $0 \leq \theta \leq 2\pi$. **[4 marks]**

(ii) Describe the curve, C, in detail. **[3 marks]**

(iii) Find the equations of the tangent and normal to the curve, C, at the point given by $\theta = 0$. **[5 marks]**

Total 25 marks

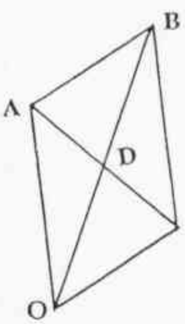
4. (a) (i) Express $\cos 4\theta$ in terms of $\cos 2\theta$. **[2 marks]**

(ii) Hence, solve the equation

$$\cos 4\theta + 3 \cos 2\theta - 1 = 0,$$

for $0 < \theta < \pi$. **[10 marks]**

(b) The diagram below shows a parallelogram, OABC, whose diagonals intersect at D.



The position vectors of A and C relative to O are $-4\mathbf{i} + 7\mathbf{j}$ and $3\mathbf{i} + 4\mathbf{j}$ respectively.

(i) Determine a unit vector in the direction of OC. **[3 marks]**

(ii) Find, by calculation, the position vector of D. **[5 marks]**

(iii) The point, E, which lies within the parallelogram, has position vector $(-8 \sin^2 \theta)\mathbf{i} + (6 - 6 \cos^2 \theta)\mathbf{j}$. Show that OE and OC are perpendicular. **[5 marks]**

Total 25 marks

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SECTION 3 (MODULE 1.3)

Answer BOTH questions.

5. (a) Let the area under the curve $y = f(x)$ represented by $\int_a^b f(x)\,dx$ be divided into n strips, each of width d units so that each strip is approximately a trapezium. With the aid of a diagram, show that for n = 4,

$$\int_a^b f(x)\,dx \approx \frac{1}{2}\,d\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right],$$

where $y_0 = f(a)$, $y_1 = f(a+d)$, ..., $y_4 = f(a+4d)$. [6 marks]

(b) (i) The length, L, of a rod decreases at the rate of $\frac{1}{2} - \frac{t}{1+t^2}$ metres per second. It was 10 metres long at time $t = 0$. Write down an appropriate differential equation connecting the length of the rod and the rate of decrease at time, t. Hence, derive an expression for L. [8 marks]

(ii) Evaluate the length, L, of the rod using the trapezium rule for five ordinates and strips of length 1 unit. [7 marks]

(iii) Find the time at which the rate of decrease vanishes. [2 marks]

(iv) Show that the rate of decrease is never negative. [2 marks]

Total 25 marks

6. The function, h, is given by

$h(x) = x^3 - 3x + 2,\ x \in$ R.

(a) Find the values of x for which $h(x) = 0$. [5 marks]

(b) Find the stationary points of h. [6 marks]

(c) Determine the value(s) of x where h has

(i) a local maximum [2 marks]

(ii) a local minimum. [2 marks]

(d) Using the above, and any other information, sketch the graph of h. [5 marks]

(e) Find the total area enclosed between the curve, the x-axis and the values $x = -2$ and $x = 2$. [5 marks]

Total 25 marks

END OF TEST