TEST CODE **000571**

FORM TP 99194

PILOT/MAY 1999

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

$1\frac{1}{2}$ hours

02 JUNE 1999 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 6 questions.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

SECTION 1 (MODULE 1.1)

Answer ALL The Questions.

1. Given that $x - 2$ is a factor of $x^3 - kx^2 + 5x + k$, calculate the value of k. **[2 marks]**

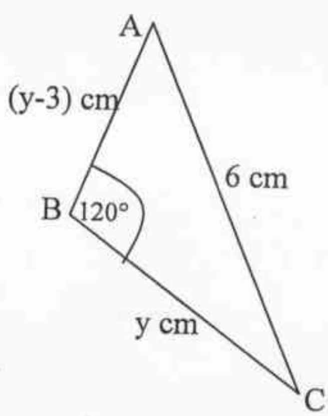
2. Find the real values of x which satisfy the equation:

   $|3x| = 2x + 5.$ **[3 marks]**

3. Express $32 - x^5$ in the form $(2 - x)\,g(x)$, where $g(x)$ is a polynomial in x. **[3 marks]**

4. Solve the equation $81^{x+1} = 27^x$. **[4 marks]**

5. In triangle ABC below, angle ABC = 120°, BA = (y-3) cm, BC = y cm and AC = 6 cm.



Show that $y^2 - 3y - 9 = 0$. **[3 marks]**

6. **The function f is given by:**

f: $x \rightarrow 2x - 3, x \in \mathbb{Z}$.

(a) **Show that f is:**

(i) **injective (1:1)** **[1 mark]**

(ii) **not surjective (not onto Z).** **[2 marks]**

(b) **Determine the value(s) of $x \in \mathbb{Z}$ for which $f^{-1}(x) \in \mathbb{Z}$.** **[2 marks]**

SECTION 2 (MODULE 1.2)

Answer ALL The Questions.

7. Given that α and β are roots of the equation $2x^2 - 4x - 5 = 0$,

find, without solving the equation, the exact value of $(\alpha^2 - \beta^2)$. **[3 marks]**

8. Prove that $\cos^2\frac{\theta}{2} - \cos\theta = \sin^2\frac{\theta}{2}$. **[3 marks]**

9. In a triangle ABC, show that $\vec{AB} + \vec{AC} = 2\,\vec{AD}$, where D is the midpoint of BC.

[3 marks]

10. Sketch a graph to represent the range of values of x which satisfy the inequality

$x^2 - 8 \leq -2x$. **[3 marks]**

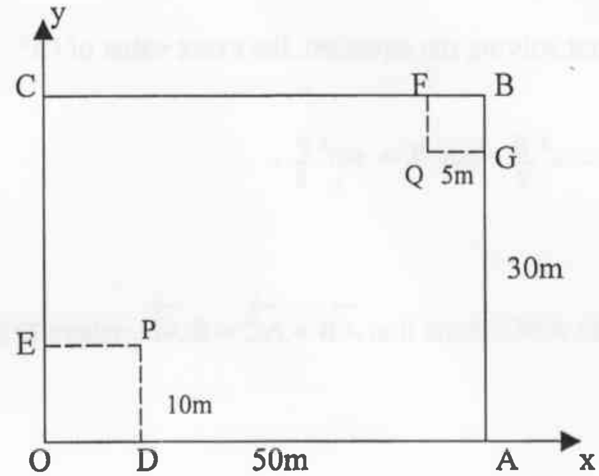
11. Given that points C (-1, 1) and D (2, -2), find the equation of

(a) the line CD **[2 marks]**

(b) the line through the E(-1,-2) parallel to the line CD. **[2 marks]**

12. In the diagram below, not drawn to scale, OABC is a horizontal rectangular playing field.

The linear dimensions of the field are 30 metres and 50 metres, respectively. ODPE and BFQG are squares shown with PD = 10m and QG = 5m.



A ball moves on the surface of the field from point P to point Q. With respect to the axes drawn on the diagram,

(a) find the coordinates of P and Q **[2 marks]**

(b) calculate the length of the straight line PQ, correct to 2 decimal places. **[2 marks]**

SECTION 3 (MODULE 1.3)

Answer ALL The Questions.

13. Given that $\lim_{x \to 4} (a\sqrt{x} + x) = 9$, calculate the value of a. **[2 marks]**

14. The function f is defined by

$$f(x) = \frac{1}{(x+2)(x-1)}$$

and is continuous for all values of x except a and b, where a< b. Find the value of a and of b. **[2 marks]**

15. (a) Find the value(s) of x at the stationary points of the function $f : x \to 2x^3 - 9x^2$.

(b) Determine the nature of the points. **[4 marks]**

16. Find, from first principles, the derivative of $3x^2$ with respect to x. **[4 marks]**

17. The function f is defined by

$$f(x) = (x^3 + x^2 - 2) / x^5, \text{ where } x \in \mathbf{R}, x \neq 0$$

(a) Find $\int f(x)dx$.

(b) Evaluate $\int_1^2 f(x)dx$. **[5 marks]**

18. Given that $\int_2^5 f(x)dx = 25$, where f is a real continuous function, evaluate

$$\int_2^5 2\{f(x) + x\}\, dx.$$ **[3 marks]**

END OF TEST