TEST CODE 000572

FORM TP 99195

PILOT/MAY 1999

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 02

2 hours

03 JUNE 1999 (a.m.)

99195

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the THREE sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Ruler and compass

SECTION 1 (MODULE 1.1)

Answer ALL The Questions.

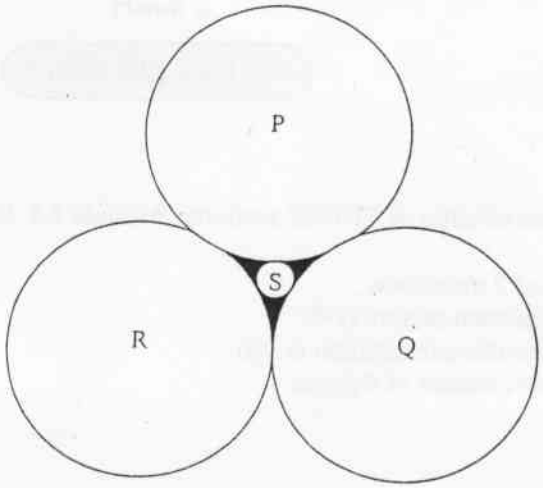
1. (a) Solve the simultaneous equations:

$$(x - 2)^2 + (y + 2)^2 = 4,$$

$$y + x - 2 = 0.$$

[8 marks]

(b)



The diagram above represents the logo of a company. The logo consists of three circles, each of radius r, which touch one another externally. P, Q and R are the centres of these three large circles. A fourth circle, with centre S and radius α, touches each of the three large circles.

(i) Write down the size of angle PQR. **[1 mark]**

(ii) Calculate, in terms of r, the area of triangle PQR. **[2 marks]**

(iii) Write down the size of angle PSQ. **[1 mark]**

By considering triangle PSR, or otherwise,

(iv) show that $\dfrac{r}{r + \alpha} = \dfrac{\sqrt{3}}{2}$ **[2 marks]**

(v) calculate, in terms of r, the area of the shaded region. **[6 marks]**

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2. The function f is defined by $f : x \rightarrow 1 - 6x - 3x^2, x \in \mathbb{R}$.

(a) (i) Evaluate $f f(-2)$. **[2 marks]**

(ii) Calculate the exact values of x which map onto themselves under the function f. **[4 marks]**

(iii) Express $f(x)$ in the form $u(x - v)^2 + w$, where $u, v, w \in \mathbb{R}$. **[3 marks]**

(iv) By sketching the graph of $y = f(x)$, or otherwise, state the turning point P of the graph and indicate whether P is a maximum or a minimum. **[3 marks]**

(b) (i) Determine the range of f. **[1 mark]**

(ii) Explain why the function f has an inverse. **[1 mark]**

(iii) Find an expression for $f^{-1}(x)$. **[4 marks]**

(iv) Describe the geometrical relationship between the graphs

$y = f(x)$ and $y = f^{-1}(x)$. **[2 marks]**

SECTION 2 (MODULE 1.2)

3. (a) (i) Find the equation of the line which passes through the point (4, -1) and is perpendicular to the straight line $y = -2x + 2$. **[3 marks]**

(ii) Calculate the coordinates of the point of intersection of these two straight lines. **[3 marks]**

(b) The function f is defined on the set **R** of real numbers by

$f : x \rightarrow (k + 3)x^2 + kx + 1$, where $k \in \mathbf{R}$.

(i) Calculate the range of values of k for which the equation $f(x) = 0$ has no real roots. **[5 marks]**

(ii) Solve the equation $f(x) = 0$ for $k = 1$, giving your answer in the form $a \pm bi$, where $a, b \in \mathbf{R}$. **[3 marks]**

(iii) Let α and β be roots of $f(x) = 0$ when $k = 8$. Without first solving $f(x) = 0$, determine the equation whose roots are respectively $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. **[6 marks]**

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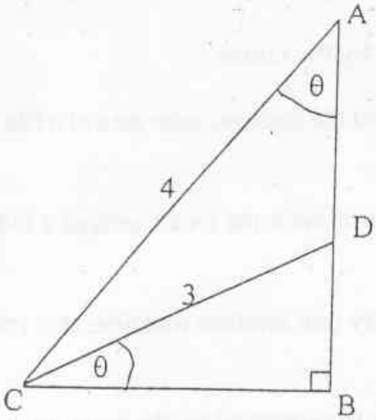
4. (a) Given that θ is an obtuse angle such that $\sin \theta = 2/3$, find the value of $\cos 2\theta$.

[2 marks]

(b)



The diagram above shows a triangle ABC in which AC = 4 units, CD = 3 units and angle CAB = angle DCB = θ. AB is perpendicular to CB.

(i) Obtain an expression for AD in terms of θ. **[3 marks]**

(ii) Express AD in the form $R\cos(\theta + \alpha)$, where R is positive and α is an acute angle.

[4 marks]

(c) (i) Express $\cos 3\theta$ in terms of $\cos \theta$. **[5 marks]**

(ii) Hence, solve, for $0 < \theta < 2\pi$, the equation

$$\cos 3\theta + 2\cos 2\theta + 4\cos \theta + 2 = 0.$$ **[6 marks]**

SECTION 3 (MODULE 1.3)

A groove cutter is operated by a robot to produce small discs from a flat sheet of metal. The path of the cutter blade traces out the curve $y = \sin t$, $o \le t \le \pi$, after t seconds.

(a) Sketch the curve traced out by the cutter. **[2 marks]**

(b) Use the Trapezium rule to find the approximate area of a flat side of each disc by using eight (8) subintervals. **[6 marks]**

(c) Compare this approximate area with the exact area of a side of each disc. **[4 marks]**

(d) Each disc is fed automatically into another machine that rotates it through 360° about the t-axis to sand the edge.

(i) Sketch the solid that is generated by the rotation. **[2 marks]**

(ii) Find the volume of the solid that is generated. **[6 marks]**

The function g is given by $g(x) = (x^2 - 1)(x + 1)$, $x \in \mathbf{R}$.

(a) Find g(0), g(1) and g(-1). **[3 marks]**

(b) Obtain an expression for the derivative $g'(x)$ of g at $x \in \mathbf{R}$. **[2 marks]**

(c) Find the stationary points of g. **[4 marks]**

(d) Determine the value(s) of x where g has

(i) a local maximum **[2 marks]**

(ii) a local minimum. **[2 marks]**

(e) Using the above, and any other information, sketch the graph of g. **[7 marks]**

END OF TEST